(2c)

$6x^3 + x^2 - 2x \neq 0$

$x(6x^2 + x - 2) \neq 0$

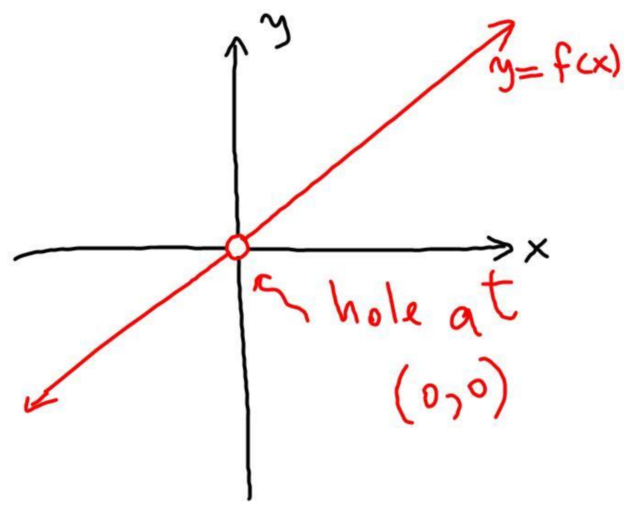
$x \neq 0$ and $x \neq \frac{-1 \pm \sqrt{1+48}}{12} = \frac{-1 \pm 7}{12}$ $\nearrow -\frac{8}{12} = -\frac{2}{3}$ $\searrow \frac{6}{12} = \frac{1}{2}$

$\therefore x \neq 0, -\frac{2}{3}, \frac{1}{2}$

(4a)

$f(x) = \frac{x^2}{x}$; $x \neq 0$

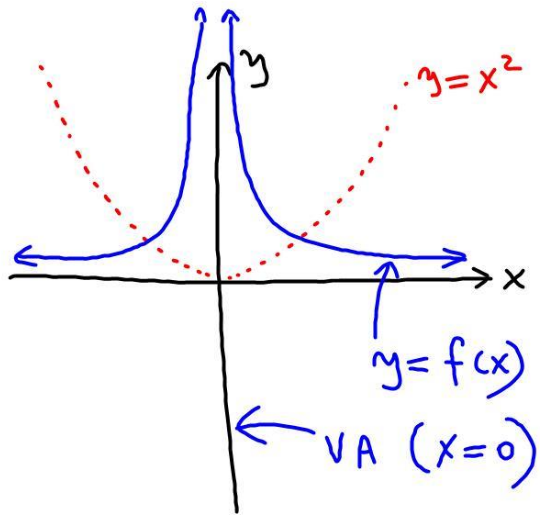
$f(x) = x$; $x \neq 0$



4b

$$f(x)=\frac{x^2}{x^4}\ ;\ x\neq 0$$

$$f(x)=\frac{1}{x^2}\ ;\ x\neq 0$$

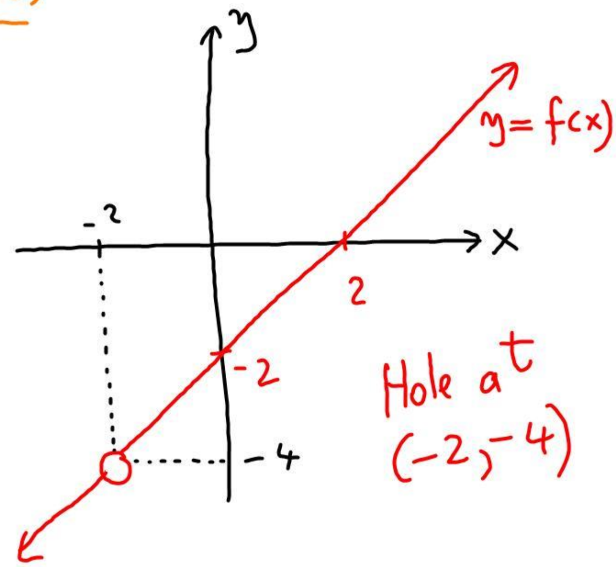


4c

$$f(x)=\frac{x^2-4}{x+2}\ ;\ x\neq -2 \quad \rightarrow \quad \frac{(x-2)\cancel{(x+2)}}{\cancel{x+2}}$$

$$f(x)=x-2\ ;\ x\neq -2$$

Simplified $f(x)$

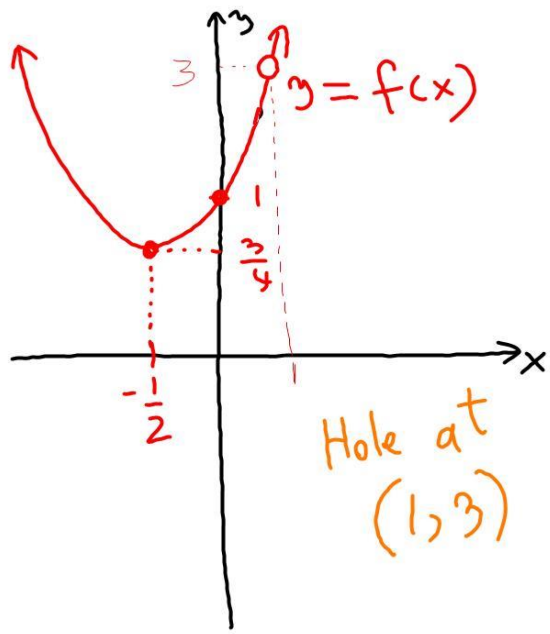


4d

$$f(x) = \frac{x^3-1}{x-1} = \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{x-1}}$$

$$f(x) = x^2+x+1 \ ; \ x \neq 1$$

$$= \left(x+\frac{1}{2}\right)^2 + \frac{3}{4}$$



Hole at (1, 3)

5a

$$f(x) = \frac{x-1}{x^2-1} = \frac{\cancel{(x-1)}}{\cancel{(x-1)}(x+1)}$$

$$f(x) = \frac{1}{x+1} \; ; \; x \neq \pm 1$$

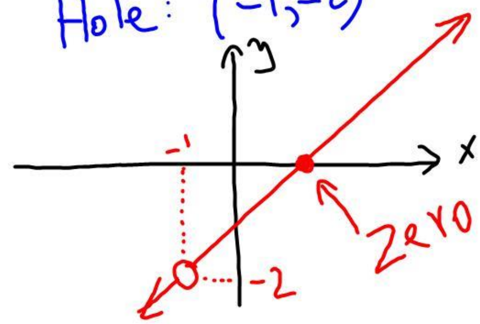
$\therefore$ no zeros

5b

$$f(x) = \frac{x^2-1}{x+1} = x-1 \; ; \; x \neq -1$$

Zeros: $x = 1$

Hole: $(-1, -2)$



6a

$$f(x) = \frac{x}{x^2+1}$$

$x^2+1=0$

no solution

VA: none

6b

$$f(x) = \frac{x+3}{x^2+x-6}$$

$$= \frac{\cancel{x+3}}{\cancel{(x+3)}(x-2)}$$

$$= \frac{1}{x-2} \; ; \; x \neq 2, -3$$

VA: $x=2$

Hole: $\left(-3, -\frac{1}{5}\right)$

Zeros: none

6c

$$f(x) = \frac{x^3-1}{x^2-1}$$

$$= \frac{\cancel{(x-1)}(x^2+x+1)}{\cancel{(x-1)}(x+1)}$$

$$= \frac{x^2+x+1}{x+1}$$

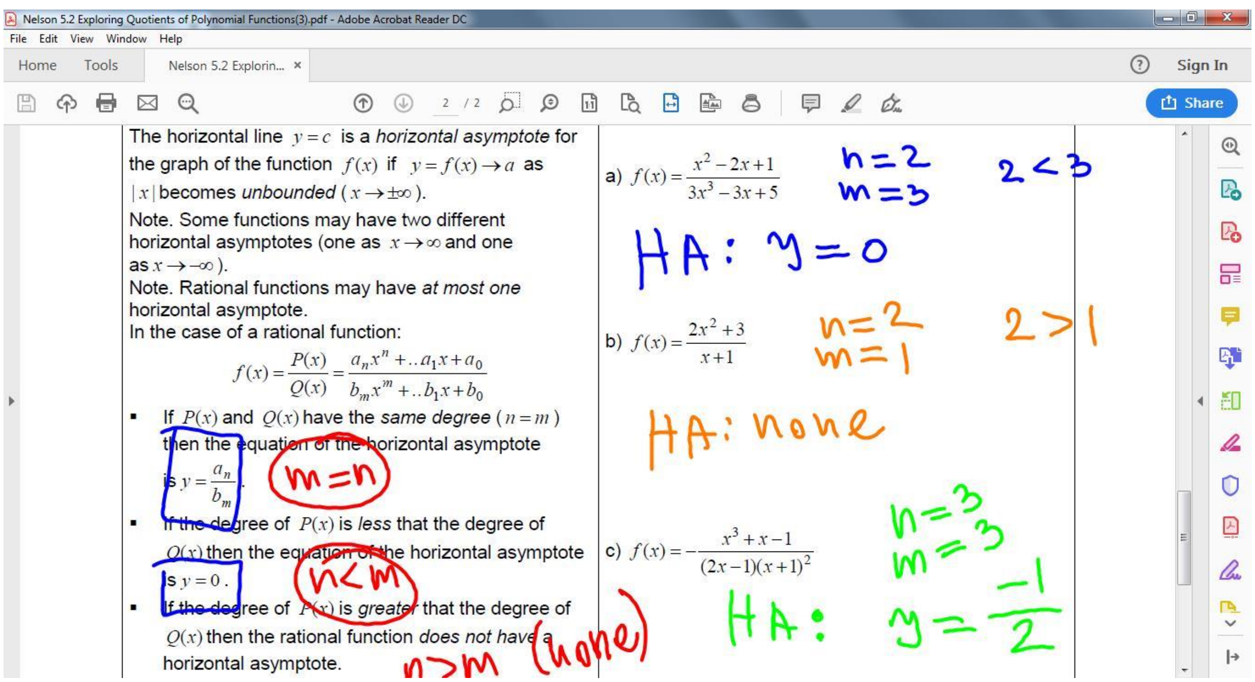
Zeros: none

VA: $x=-1$

Hole: $\left(1, \frac{3}{2}\right)$

D: $x \neq \pm 1$

$y_{int} = 1$



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The horizontal line $y=c$ is a *horizontal asymptote* for the graph of the function $f(x)$ if $y=f(x)\to a$ as $|x|$ becomes *unbounded* ($x\to\pm\infty$).

Note. Some functions may have two different horizontal asymptotes (one as $x\to\infty$ and one as $x\to-\infty$).

Note. Rational functions may have *at most one* horizontal asymptote.

In the case of a rational function:

$$f(x)=\frac{P(x)}{Q(x)}=\frac{a_n x^n+..a_1x+a_0}{b_m x^m+..b_1x+b_0}$$

- If $P(x)$ and $Q(x)$ have the *same degree* ($n=m$) then the equation of the horizontal asymptote is $\boxed{y=\frac{a_n}{b_m}}$. $m=n$
- If the degree of $P(x)$ is *less* that the degree of $Q(x)$ then the equation of the horizontal asymptote is $\boxed{y=0}$. $n<m$
- If the degree of $P(x)$ is *greater* that the degree of $Q(x)$ then the rational function *does not have* a horizontal asymptote. $n>m$ (none)

a) $f(x)=\frac{x^2-2x+1}{3x^3-3x+5}$

$n=2$
$m=3$

$2<3$

HA: $y=0$

b) $f(x)=\frac{2x^2+3}{x+1}$

$n=2$
$m=1$

$2>1$

HA: none

c) $f(x)=-\frac{x^3+x-1}{(2x-1)(x+1)^2}$

$n=3$
$m=3$

HA: $y=-\frac{1}{2}$

8a

$$f(x)=\frac{x-1}{x^2-1}$$

D: $x \neq \pm 1$

$y_{int}=f(0)=1$

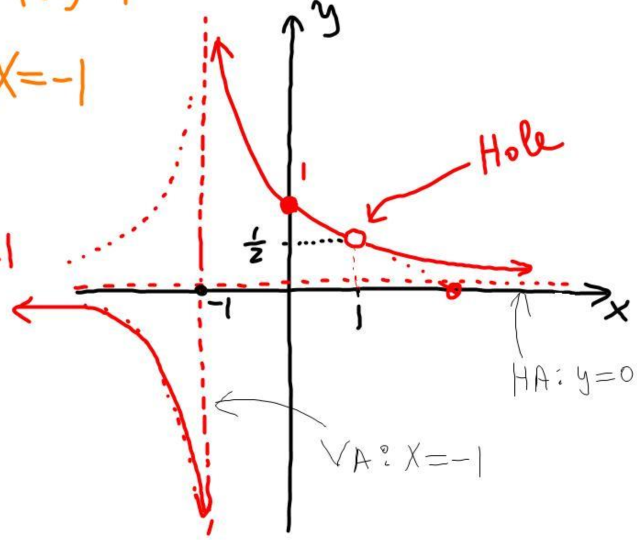
VA: $x=-1$

$$f(x)=\frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}=\frac{1}{x+1}\,;\ x\neq \pm 1$$

Hole: $\left(1, \frac{1}{2}\right)$

Zeros: none

HA: $y=0$



8b

$$f(x)=\frac{x^2-1}{x^2-4}$$

D: $x \neq \pm 2$

Zeros: $x=\pm 1$

VA: $x=\pm 2$

Holes: none

HA: $y=1$

$y_{int}=\frac{1}{4}$