4.3 Solving Polynomial Inequalities

A Sign Chart Method Use a *sign chart* to specify the sign of each factor and then combine them to find the sign of the whole factored polynomial.	Ex 1. Use a sign chart to solve each inequality. a) $x(x-1) \le 0$ b) $(x^2 - x - 2)(x^2 + 1)(x-3)^3 \ge 0$
B Substitution Method To find the sign of the polynomial expression on each interval, chose conveniently a number in that interval and evaluate the function.	Ex 2. Use a substitution to solve the following inequality. $x^2(x^2 - 4)(x+1) > 0$
C Graphical Method Graph the factored polynomial and then conclude about its sign.	Ex 3. Use the graphical method to solve each inequality. a) $x(x-1)(x+2) > 0$ b) $(x^4 - 1)(x^2 - 9) \le 0$

D Algorithm to Solve Polynomial Inequalities In order to solve an inequality involving a polynomial expression: ▪ Move all terms to one side of inequality ▪ Factor the polynomial ▪ Use the sign chart or the graphical method to find the sign of the polynomial ▪ Write the solution set	Ex 4. Solve for x. a) $(x+1)^2 \le (x-2)^3+5$ b) $(x-1)^4+8x>4(x^2+1)$
E Technology When the polynomial inequality is not factorable, use technology to find the solution set.	Ex 5. A box with an open top is to be constructed from a square piece of cardboard, $2\,m$ wide, by cutting out a square, of side length x, from each of the four corners and bending up the sides. Find x such that the volume of the box is between $0.3\,m^3$ and $0.5\,m^3$.

Reading: Nelson Textbook, Pages 219-225
Homework: Nelson Textbook, Page 226: #5, 6d, 7df, 8, 9, 10, 11, 17, 18

(1a) $\overbrace{x(x-1)}^{P(x)} \le 0$

x		0		1	
x	$-$	0	$+$	$+$	$+$
$x-1$	$-$	$-$	$-$	0	$+$
$P(x)$	$+$	0	$-$	0	$+$

$\therefore \quad x \in [0,1]$

(1b) $\overbrace{(x^2-x-2)(x^2+1)(x-3)^3}^{P(x)} \ge 0$

$(x-2)(x+1)(x^2+1)(x-3)^3 \ge 0$

sign chart

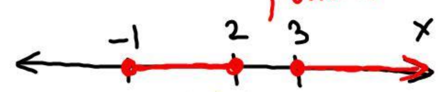
x		-1		2		3	
$x-2$	$-$	$-$	$-$	0	$+$	$+$	$+$
$x+1$	$-$	0	$+$	$+$	$+$	$+$	$+$
x^2+1	$+$	$+$	$+$	$+$	$+$	$+$	$+$
$(x-3)^3$	$-$	$-$	$-$	$-$	$-$	0	$+$
$P(x)$	$-$	0	$+$	0	$-$	0	$+$

$P(x) \ge 0$

"OR"

$\therefore x \in [-1,2] \cup [3,\infty)$

union

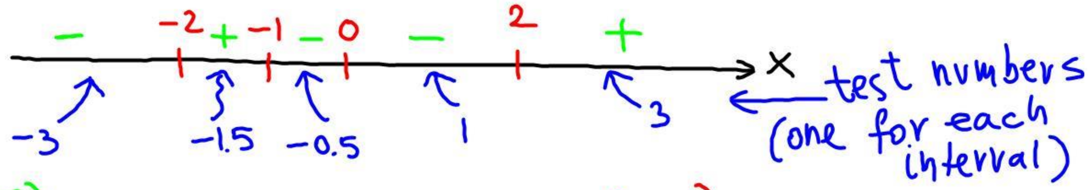


$\therefore -1 \le x \le 2$ or $x \ge 3$

② $\overbrace{x^2(x^2-4)(x+1)}^{P(x)} \geq 0 \longrightarrow x \in [-2,-1] \cup \{0\} \cup [2,\infty)$

$x^2(x-2)(x+2)(x+1) > 0$

all zeros of $P(x)$

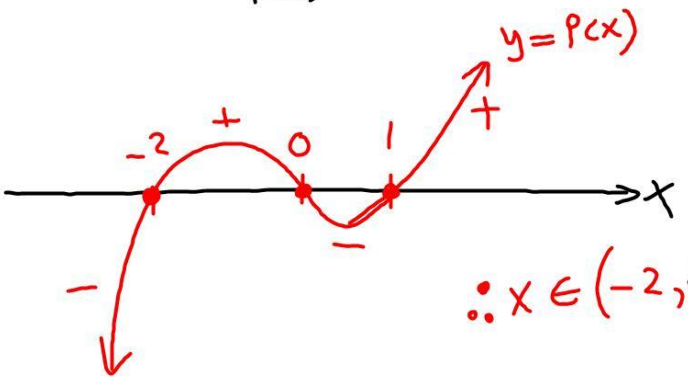


$P(-3) = -90$

$\therefore x \in (-2,-1) \cup (2,\infty)$

$-2 < x < -1$ or $x > 2$

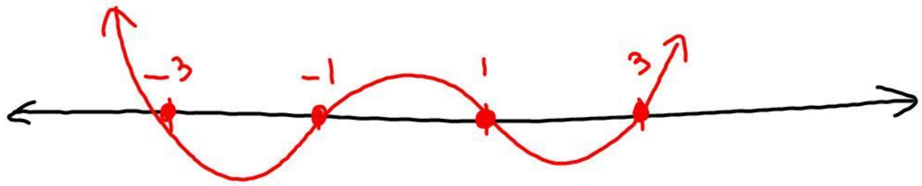
③a $\underbrace{x(x-1)(x+2)}_{P(x)} > 0$



$\therefore x \in (-2,0) \cup (1,\infty)$

3b) $(x^4-1)(x^2-9) \le 0$

$(x-1)(x+1)(x^2+1)(x-3)(x+3) \le 0$



$\therefore x \in [-3,-1] \cup [1,3]$

4a $(x+1)^2 \le (x-2)^3 + 5$

$$x^2+2x+1 \le (x-2)^2(x-2)+5$$

$$x^2+2x+1 \le (x^2-4x+4)(x-2)+5$$

$$x^2+2x+1 \le x^3-4x^2+4x-2x^2+8x-8+5$$

$$x^3-7x^2+10x-4 \geq 0$$

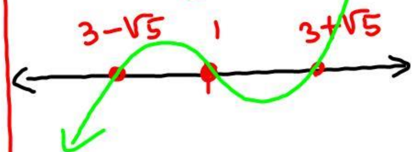
$\{\pm 1, \pm 2, \pm 4\}$

Zeros: 1,

$$\begin{array}{r|rrrr} 1 & 1 & -7 & 10 & -4 \\ & 0 & 1 & -6 & 4 \\ \hline 1 & 1 & -6 & 4 & 0 \end{array}$$

$$x^2-6x+4=0$$

$$x = \frac{6 \pm \sqrt{36-16}}{2} = \frac{6 \pm 2\sqrt{5}}{2} = 3 \pm \sqrt{5}$$



$\therefore [3-\sqrt{5}, 1] \cup [3+\sqrt{5}, \infty)$

4b $(x-1)^4 + 8x > 4(x^2+1)$

$$(x^2-2x+1)(x^2-2x+1)+8x > 4x^2+4$$

$$x^4-2x^3+x^2-2x^3+\cancel{4x^2}-2x+x^2-2x+1+8x > \cancel{4x^2}+4$$

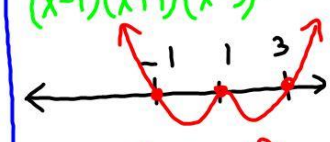
$$x^4-4x^3+2x^2+4x-3>0$$

$\{\pm 1, \pm 3\}$

Zeros: 1, −1

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 2 & 4 & -3 \\ & 0 & 1 & -3 & -1 & 3 \\ \hline -1 & 1 & -3 & -1 & 3 & 0 \\ & 0 & -1 & 4 & -3 & \\ \hline & 1 & -4 & 3 & 0 & \end{array}$$

$$(x-1)^2(x+1)(x-3)>0$$



$\therefore x \in (-\infty, -1) \cup (3, \infty)$