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## 4.3 Solving Polynomial Inequalities

### A Sign Chart Method

Use a *sign chart* to specify the sign of each factor and then combine them to find the sign of the whole factored polynomial.

(1a) $\underbrace{x(x-1)}_{P(x)} \le 0 \iff P(x) \le 0$

| x | $(-\infty, 0)$ | 0 | $(0,1)$ | 1 | $(1, \infty)$ |
|---|---|---|---|---|---|
| x | − | 0 | + | + | + |
| $x-1$ | − | − | − | 0 | + |
| $P(x)$ | + | 0 | − | 0 | + |

$\therefore x \in [0, 1]$

**Ex 1.** Use a sign chart to solve each inequality.

a) $x(x-1) \le 0$

b) $(x^2 - x - 2)(x^2 + 1)(x-3)^3 \ge 0$

### B Substitution Method

To find the sign of the polynomial expression on each interval, chose conveniently a number in that interval and evaluate the function.

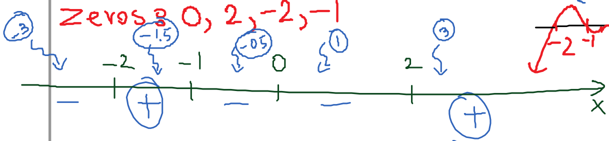
Intervals to check

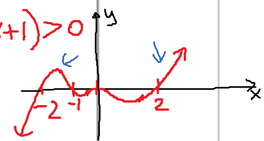
| | |
|---|---|
| $x < -2$ | $P(3) > 0$ |
| $-2 < x < -1$ | $P(1) < 0$ |
| $-1 < x < 0$ | ...... |
| $0 < x < 2$ | |
| $x > 2$ | |

**Ex 2.** Use a substitution to solve the following inequality.

$\underbrace{x^2(x^2-4)(x+1)}_{P(x)} > 0 \Rightarrow x^2(x-2)(x+2)(x+1) > 0$

Zeros: 0, 2, −2, −1



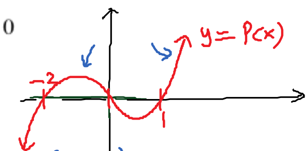

$\boxed{P(x) > 0}$ $\therefore x \in (-2, -1) \cup (2, \infty)$

### C Graphical Method

Graph the factored polynomial and then conclude about its sign.

**Ex 3.** Use the graphical method to solve each inequality.

a) $\underbrace{x(x-1)(x+2)}_{P(x)} > 0$

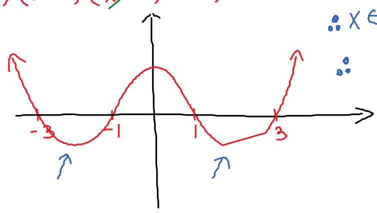



$\boxed{P(x) > 0}$

$\therefore x \in (-2, 0) \cup (1, \infty)$

b) $(x^4 - 1)(x^2 - 9) \le 0$

$(x-1)(x+1)(\cancel{x^2+1})(x-3)(x+3) \le 0$ $\quad x^2 + 1 > 0$




$\therefore x \in [-3, -1] \cup [1, 3]$

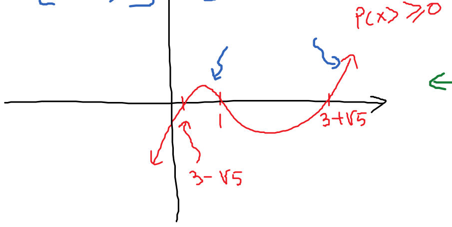
$\therefore -3 \le x \le -1$ OR $1 \le x \le 3$

## D Algorithm to Solve Polynomial Inequalities

In order to solve an inequality involving a polynomial expression:

- Move all terms to one side of inequality
- Factor the polynomial
- Use the sign chart or the graphical method to find the sign of the polynomial
- Write the solution set

$\therefore x \in [3-\sqrt{5}, 1] \cup [3+\sqrt{5}, \infty)$




## Ex 4. Solve for $x$.

a) $(x+1)^2 \le (x-2)^3 + 5$

$(a+b)^2 = a^2+2ab+b^2$
$(a-b)^2 = a^2-2ab+b^2$

$(x-2)^3 + 5 - (x+1)^2 \geq 0$

$(x-2)(x-2)^2 + 5 - (x^2+2x+1) \geq 0$

$(x-2)(x^2-4x+4) + 5 - x^2 - 2x - 1 \geq 0$

$x^3 - 4x^2 + 4x - 2x^2 + 8x - 8 + 5 - x^2 - 2x - 1 \geq 0$

$\underbrace{x^3 - 7x^2 + 10x - 4}_{P(x)} \geq 0$

$\pm 1, \pm 2, \pm 4$

$P(1) = 1 - 7 + 10 - 4 = 0$ ✓

$$\begin{array}{c|cccc} 1 & 1 & -7 & 10 & -4 \\ & 0 & 1 & -6 & 4 \\ \hline & 1 & -6 & 4 & 0 \end{array}$$

$x^2 - 6x + 4 = 0$

$ax^2 + bx + c = 0$

$x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \dfrac{6 \pm \sqrt{36-16}}{2}$

$x = \dfrac{6 \pm 2\sqrt{5}}{2}$

$x = 3 \pm \sqrt{5}$

b) $(x-1)^4 + 8x > 4(x^2+1)$

## E Technology

When the polynomial inequality is not factorable, use technology to find the solution set.

## Ex 5.

A box with an open top is to be constructed from a square piece of cardboard, $2\,m$ wide, by cutting out a square, of side length $x$, from each of the four corners and bending up the sides. Find $x$ such that the volume of the box is between $0.3\,m^3$ and $0.5\,m^3$.

**Reading**: Nelson Textbook, Pages 219-225
**Homework**: Nelson Textbook, Page 226: #5, 6d, 7df, 8, 9, 10, 11, 17, 18




1b) $3(x^2-x-2)(x^2+1)(x-3)^3 \geq 0$

$x^2+1 \geq 1$

$\cancel{3}(x-2)(x+1)\cancel{(x^2+1)}(x-3)^3 \geq 0$ (factorize)

$\underbrace{(x-2)(x+1)(x-3)^3}_{P(x)} \geq 0 \Rightarrow \boxed{P(x) \geq 0}$ (simplify) $\div 3$ | $\div (x^2+1)$

| x | $(-\infty,-1)$ | $-1$ | $(-1,2)$ | 2 | $(2,3)$ | 3 | $(3,\infty)$ |
|---|---|---|---|---|---|---|---|
| $x-2$ | − | − | − | 0 | + | + | + |
| $x+1$ | − | 0 | + | + | + | + | + |

What are the intervals to check:
$(-\infty,-1)$, $(-1,2)$, $(2,3)$, $(3,\infty)$
$x<-1$, $-1<x<2$, $2<x<3$, $x>3$

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $x-2$ | $-$ | $-$ | $-$ | 0 | $+$ | $+$ | $+$ |
| $x+1$ | $-$ | 0 | $+$ | $+$ | $+$ | $+$ | $+$ |
| $(x-3)^3$ | $-$ | $-$ | $-$ | $-$ | $-$ | 0 | $+$ |
| $P(x)$ | $-$ | 0 | $+$ | 0 | $-$ | 0 | $+$ |

$(-\infty,-1)$, $(-1,2)$, $(2,3)$, $(3,\infty)$

$x<-1$, $-1<x<2$, $2<x<3$, $x>3$

| $x$ | $\boxed{0}$ | 2 | |
|---|---|---|---|
| $2-x$ | $+$ | 0 | $-$ |

$\therefore\ x \in [-1,2] \cup [3,\infty)$

union

$\therefore\ -1 \le x \le 2$ OR $x \ge 3$

$\cup \equiv$ OR