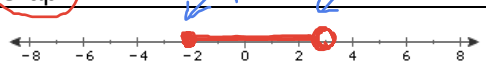
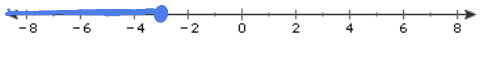
4.2 Solving Linear Inequalities

A Interval Notation

(a,b) means $a < x < b$ (x is *greater than* a and is *less than* b)
$[a,b]$ means $a \le x \le b$ (x is *greater or equal to* a and is *less or equal to* b)
$(a,b]$ means $a < x \le b$ (x is *greater than* a and is *less or equal to* b)
(a,∞) means $x > a$ (x is *greater than* a)
$(-\infty,a]$ means $x \le a$ (x is *less or equal to* a)

Ex 1. Complete the following table.

Inequality Notation	Solution Set Notation	Interval Notation	Solution Set Graph
$-2 \le x < +3$	$\{x \in \mathbb{R} \mid -2 \le x < 3\}$	$[-2,3)$	
$x \le -3$	$\{x \in R \mid x \le -3\}$	$(-\infty,-3]$	
$x > -4$	$\{x \in \mathbb{R} \mid x > -4\}$	$(-4,\infty)$	

B Inequalities

The inequality symbols: < (*less*), $\le$ (*less or equal to*), > (*greater than*), $\ge$ (*greater or equal to*), and $\ne$ (*not equal to*) are used to create *inequalities*.

The *solution set* is the set of all numbers that make the inequality a *true statement*.

Ex 2. Verify if the given number is a solution for the given inequality.

a) $-2x+1<0$, $x=0$

b) $x-2<x^2$, $x=2$

c) $\dfrac{3-x}{x} < -3$, $x=-1$

C Inequality properties

The inequality $a < b$ is equivalent to:

$(i)\ a+c<b+c$
$(ii)\ ac<bc,\ for\ c>0$
$(iii)\ ac>bc,\ for\ c<0$

Ex 3. Solve each inequality.

a) $-2x+3<5-3x$

b) $2-3(x-1) \ge 2(3-x)-4$

Ex 4. Solve each inequality.

a) $\frac{x}{2} \geq \frac{1}{4} + \frac{x}{3}$

b) $x^2 - 1 \leq (x+1)^2$

c) $(x+2)^2 > (x-2)^2$

d) $(1+x)^2 - (1-x)^2 \geq 1$

D Simultaneous (Double) Inequality

The *simultaneous inequality* $a < x \leq b$ is equivalent to:

$$a < x \text{ and } x \leq b$$

c) $\frac{x}{2} \geq \frac{x}{3} \geq 1 - x$

Ex 5. Solve each inequality. Graph the solution set.

a) $4 > 2(x+3) > 0$

b) $x + 1 \leq 7 - 2x < -x + 6$

Ex 6. Solve each inequality. Graph the solution set.

a) $x \leq x + 1 \leq x - 2$

b) $x + 2 \geq x + 1 \geq x$

c) $0 \leq x + 1 \leq -2$

d) $1 \geq \frac{x}{2} - 3 \geq 1$

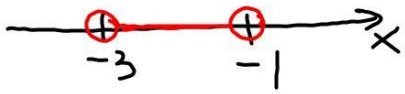
Reading: Nelson Textbook, Pages 207-212
Homework: Nelson Textbook, Page 213: #1e, 2f, 3, 4cf, 5e, 6f, 7f, 9, 12, 15, 17

(5a) $4 > 2(x+3) > 0$

$4 > 2(x+3)$ and $2(x+3) > 0$

$-2x > 6-4$ and $2x > -6$

$-2x > 2$ and $x > -3$



$x < -1$ and $x > -3$

$\therefore -3 < x < -1$ $\quad \therefore x \in (-3, -1)$

(5b) $x+1 \leq 7-2x < -x+6$

$x+1 \leq 7-2x$ and $7-2x < -x+6$

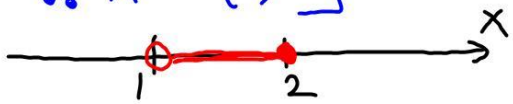
$x+2x \leq 7-1$ and $-2x+x < 6-7$

$3x \leq 6$ and $-x < -1$

$x \leq 2$ and $x > 1$

$\therefore x > 1$ and $x \leq 2$ $\quad \therefore x \in (1, 2]$

$\therefore 1 < x \leq 2$



(5c) $\frac{x}{2} \geq \frac{x}{3} \geq 1-x$

$\frac{x}{2} \geq \frac{x}{3}$ $|\cdot 6$ and $\frac{x}{3} \geq 1-x$ $|\cdot 3$

$3x \geq 2x$ and $x \geq 3-3x$

$x \geq 0$ and $4x \geq 3$

$x \geq 0$ and $x \geq \frac{3}{4}$

$\therefore x \geq \frac{3}{4}$

(3b)

$2-3(x-1) \geq 2(3-x)-4$

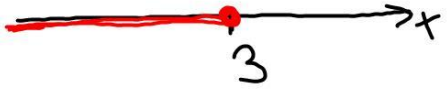
expand the brackets: $2-3x+3 \geq 6-2x-4$

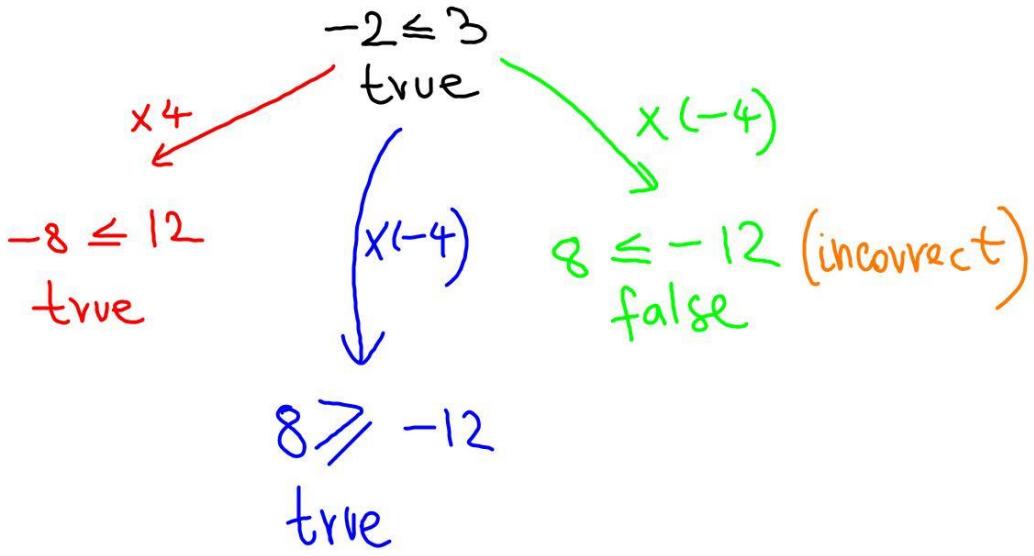
isolate the variable: $-3x+2x \geq 6-4-2-3$

simplify: $-x \geq -3$ $|\cdot(-1)$ → or $+3 \geq x$

Conclude: $\therefore x \leq 3$ $\therefore x \leq 3$

graph:





(6a) $x \leq x+1 \leq x-2$

$x \leq x+1$ and ~~$x+1 \leq x-2$~~

$0 \leq 1$ and $3 \leq 0$

true and false

$\therefore$ no solution

$\therefore x \in \emptyset$ ← empty set