4.2 Solving Linear Inequalities

A Interval Notation

(a, b) means \( a < x < b \) (\( x \) is greater than \( a \) and is less than \( b \))

\([a, b]\) means \( a \leq x \leq b \) (\( x \) is greater or equal to \( a \) and is less or equal to \( b \))

(a, b] means \( a < x \leq b \) (\( x \) is greater than \( a \) and is less or equal to \( b \))

(a, \( -\infty \)) means \( x > a \) (\( x \) is greater than \( a \))

(\( -\infty, a \)) means \( x \leq a \) (\( x \) is less or equal to \( a \))

Ex 1. Complete the following table.

<table>
<thead>
<tr>
<th>Inequality Notation</th>
<th>Solution Set</th>
<th>Interval Notation</th>
<th>Solution Set Graph</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-2 \leq x \leq 3)</td>
<td>{x \in \mathbb{R} \mid x \leq -3}</td>
<td>(-4, ( \infty ))</td>
<td><img src="image1" alt="Graph" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><img src="image2" alt="Graph" /></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td><img src="image3" alt="Graph" /></td>
</tr>
</tbody>
</table>

B Inequalities

The inequality symbols: \(< \) (less), \(\leq\) (less or equal to), \(>\) (greater than), \(\geq\) (greater or equal to), and \(\neq\) (not equal to) are used to create inequalities.

The solution set is the set of all numbers that make the inequality a true statement.

Ex 2. Verify if the given number is a solution for the given inequality.

a) \(-2x + 1 < 0\), \(x = 0\)

b) \(x - 2 < x^2\), \(x = 2\)

c) \(\frac{3-x}{x} < -3\), \(x = -1\)

C Inequality properties

The inequality \(a < b\) is equivalent to:

(i) \(a + c < b + c\)

(ii) \(ac < bc\), for \(c > 0\)

(iii) \(ac > bc\), for \(c < 0\)

Ex 3. Solve each inequality.

a) \(-2x + 3 < 5 - 3x\)

b) \(2 - 3(x - 1) \geq 2(3 - x) - 4\)
Ex 4. Solve each inequality.

| a) \( \frac{x}{2} \geq \frac{1}{4} + \frac{x}{3} \) | c) \((x + 2)^2 > (x - 2)^2\) |
| b) \(x^2 - 1 \leq (x + 1)^2\) | d) \((1 + x)^2 - (1 - x)^2 \geq 1\) |

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**D Simultaneous (Double) Inequality**

The simultaneous inequality \(a < x \leq b\) is equivalent to:

\[a < x \text{ and } x \leq b\]

| c) \(\frac{x}{2} \geq \frac{x}{3} \geq 1 - x\) | Ex 5. Solve each inequality. Graph the solution set. |
| b) \(x + 1 \leq 7 - 2x < -x + 6\) | a) \(4 > 2(x + 3) > 0\) |

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Ex 6. Solve each inequality. Graph the solution set.

| a) \(x \leq x + 1 \leq x - 2\) | c) \(0 \leq x + 1 \leq -2\) |
| b) \(x + 2 \geq x + 1 \geq x\) | d) \(1 \geq \frac{x}{2} - 3 \geq 1\) |

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**Reading:** Nelson Textbook, Pages 207-212  
**Homework:** Nelson Textbook, Page 213: #1e, 2f, 3, 4cf, 5e, 6f, 7f, 9, 12, 15, 17