4.1 Solving Polynomial Equations

A Polynomial Equation
A polynomial equation is defined as:

\[ P(x) = 0 \]

where \( P(x) \) is a polynomial function.

Note. The numbers \( x \) satisfying the polynomial equation are called the roots or the solutions of the polynomial equation.

Note. The roots (solutions) of the polynomial equation \( P(x) = 0 \) are the same as the zeros of the polynomial function \( y = P(x) \).

Ex 1. Show that \( x = \sqrt{3} \) is a solution of the polynomial equation

\[ x^4 + 9 = 6x^2 \]

B Grouping
Some polynomial equations may be solved by grouping terms adequately.

Note. Is not easy to see how to group terms in order to solve the equation.

Ex 2. Solve for \( x \) by grouping.

\[ 8x^3 - 12x^2 - 2x + 3 = 0 \]

C Integral Zeros Theorem
If \( x = b \) is an integral zero of the polynomial \( P(x) \) with integral coefficients, then \( b \) is a factor (divisor) of the constant term \( a_0 \) of the polynomial.

Note. A real zeros of the polynomial function \( P(x) \) is also called \( x \)-intercept because the graph touches or crosses the \( x \)-axis at this number.

Ex 3. Solve for \( x \) by looking first at integral roots (solutions).

\[ (x^2 - 13)x = 15 - 3x^2 \]
### D Rational Zero Theorem

If \( x = \frac{b}{a} \) is an **rational zero** of the polynomial \( P(x) \) with integral coefficients, then \( b \) is a **factor (divisor)** of the **constant term** \( a_0 \) and \( a \) is a **factor (divisor)** of the **leading coefficient** \( a_n \).

<table>
<thead>
<tr>
<th>Ex 4.</th>
<th>Solve for ( x ) by looking first at rational roots (solutions).</th>
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<tbody>
<tr>
<td></td>
<td>( 3x^4 + \frac{7}{2}x^3 - \frac{2}{3}x^2 - \frac{3}{2}x - \frac{1}{3} = 0 )</td>
</tr>
</tbody>
</table>

| Ex 5. Is \(-\frac{2}{3}\) a possible rational zero of \( P(x) = 2x^3 + ... + -8 \)? Explain. |

### E Non Rational Zeros

If \( x = a + b\sqrt{c} \) (\( a, b, c \) are rational numbers, \( c > 0 \)) is a zero of a polynomial with integral coefficients \( P(x) \), then \( x = a - b\sqrt{c} \) is also a zero of this polynomial.

<table>
<thead>
<tr>
<th>Ex 6.</th>
<th>Solve for ( x )</th>
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<tr>
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<td>( x^4 - 2x^3 - 4x^2 + 6x + 3 = 0 )</td>
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| Ex 6. | Given that \( x = 1 + \sqrt{2} \) is one of its roots. |

### F Technology

In some cases the real roots of a polynomial equation may be found only using **technology**.

<table>
<thead>
<tr>
<th>Ex 7. How many real roots does the equation given below have?</th>
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<td>( x^3 - x^2 - 1 = 0 )</td>
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| Ex 7. | Use technology to find (real) \( x \).

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**Reading**: Nelson Textbook, Pages 196-204

**Homework**: Nelson Textbook, Page 204: #6ad, 7bf, 8a, 9b, 10, 13, 15, 16, 18, 19

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