4.1 Solving Polynomial Equations

1. Solve the following polynomial equations.

a) $x^3 + 4x^2 + x - 6 = 0$

$(x-1)(x+2)(x+3) = 0$

$\therefore x = 1, -2, -3$

b) $x^3 - 2x^2 - 4x + 8 = 0$

$(x-2)^2(x+2) = 0$

$\therefore x = 2, -2$

c) $x^4 - 6x^2 + 8x - 3 = 0$

$(x-1)^3(x+3) = 0$

$\therefore x = 1, -3$

d) $-x^4 + 5x^2 - 6 = 0$

$-(x-\sqrt{2})(x+\sqrt{2})(x-\sqrt{3})(x+\sqrt{3}) = 0$

$\therefore x = \pm\sqrt{2}, \pm\sqrt{3}$

e) [illegible]

$x - 1$

$2x^3 + x^2 - x + 3 = 0$

$(2x+3)(x^2 - x + 1) = 0$

$\therefore x = -\frac{3}{2}$

2. Find the point(s) of intersection between the graphs of the polynomial functions $f(x) = x^4 + 4$ and $g(x) = 9x^2 - 4$. Make a diagram.

$f(x) = g(x)$

$x^4 + 4 = 9x^2 - 4$

$x^4 - 9x^2 + 8 = 0$

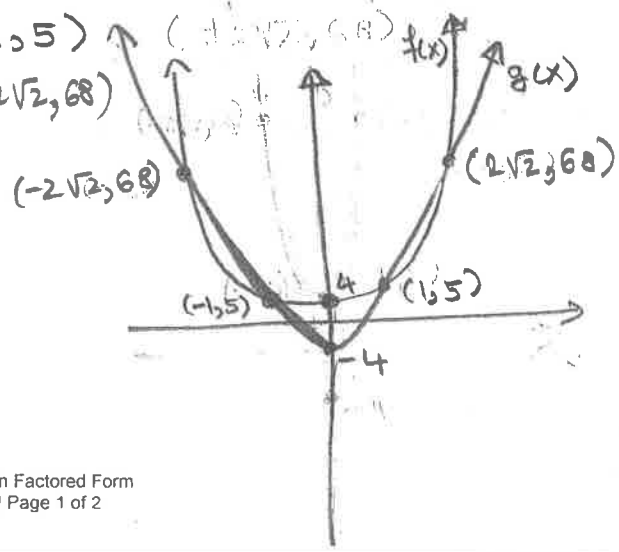
$(x-1)(x+1)(x^2 - 8) = 0$

$x = 1; x = -1; x = 2\sqrt{2}; x = -2\sqrt{2}$

$y = 5; y = 5; y = 68; y = 68$

$\therefore (\pm 1, 5)$

$(\pm 2\sqrt{2}, 68)$



3. Solve the following polynomial equations.

a) $x^2(3-x)=4$

$P(x) = -(x-2)^2(x+1)$

$\therefore \quad x = -1, 2$

b) $x^4 = x^2 + 6$

$P(x) = (x^2+2)(x^2-3)$

$\therefore \quad x = \pm\sqrt{3}$

c) $\frac{3x+1}{x^2+1} = \frac{x-1}{2}$

$(x-3)(x+1)^2 = 0$

$\therefore \quad x = 3, -1$

d) $x + \frac{1}{x} - \frac{2}{x^2} = 0$

$\frac{(x-1)(x^2+x+2)}{x^2} = 0 \qquad \therefore \quad x = 1$

e) $x^3(x+2) = 2(x^2-1)+3x$

$(x-1)(x+2)(x^2+x-1) = 0 \qquad \therefore \quad x = 1, -2, \frac{-1\pm\sqrt{5}}{2}$

f) $x(x+1)(x^4-1) = 6(x+1)^2(x^2+1)$

$(x-3)(x+1)^2(x+2)(x^2+1) = 0 \qquad \therefore \quad x = 3, -1, -2$

4. Show that the polynomial function $P(x) = x^4 - x^2 + 1$ does not have any x-intercept.

$P(x) = 0$

$x^4 - x^2 + 1 = 0$

$x^2 = \frac{1 \pm \sqrt{1-4}}{2}$

x^2 is not a real number

x is not a real number

$\therefore$ no x-intercept

