MHF4U, ADVANCED FUNCTIONS TEST 1 Chapters 3 and 4 Teacher: Mr. Gugoiu	Name SOLUTIONS Date Sep 8, 2016	$\frac{\quad}{40}$ =

1. Match the polynomial function from the left with a graph on the right. Some polynomial functions may not have a corresponding graph. [KU 4]

a) $P(x) = x(x-1)(x+2)$

b) $P(x) = x^2(x+1)$ I

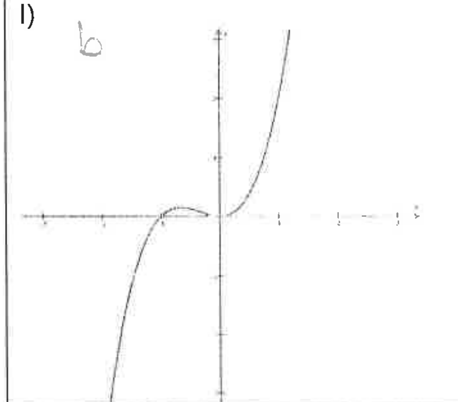
c) $P(x) = (x+1)(x-2)$ III

d) $P(x) = (x-1)(x+2)^2$

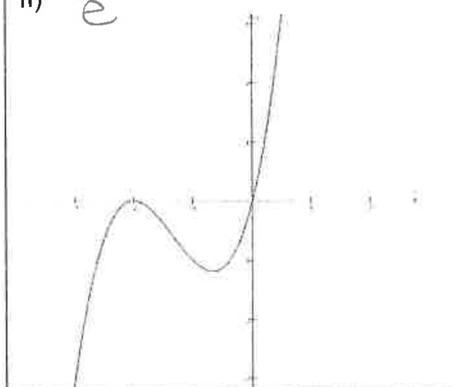
e) $P(x) = x(x+2)^2$ II

f) $P(x) = (x-1)^2(x+2)^2$ IV

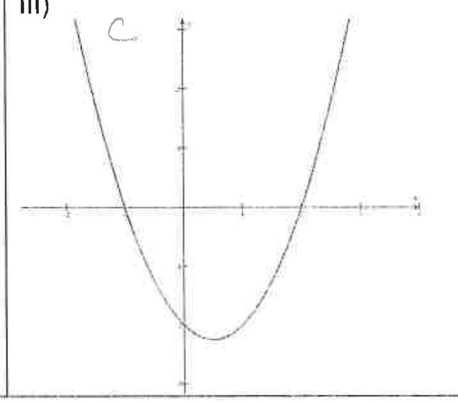
I) b



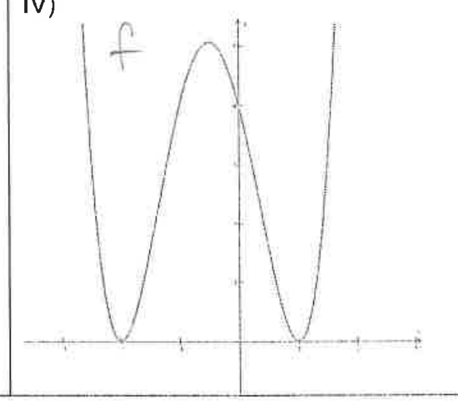
II) e



III) c



IV) f



2. Solve the following linear inequality. Show your work. [KU 2]

$2x \le 3(1-2x)+5$

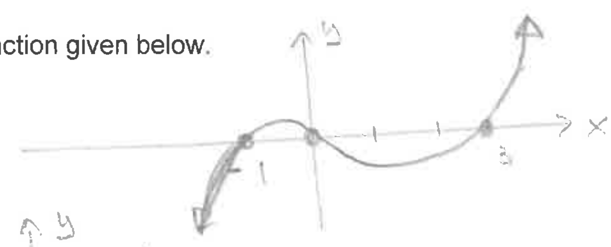
$2x \le 3 - 6x + 5$

$8x \le 8$

$\therefore x \le 1$

3. Sketch the graph of each polynomial function given below. [KU 4]

a) $P(x) = x(x+1)(x-3)$



b) $P(x) = x^2(x-1)(x+2)^3$

$\overline{10}$

4. Classify each polynomial function as even, odd, or either. Explain your conclusion. [C 2]

a) $P(x) = (x^2 - 1)^3$

$P(-x) = [(-x)^2 - 1]^3 = (x^2 - 1)^3 = P(x)$ $\therefore$ even

b) $P(x) = (x^3 - 1)^2$

$P(-x) = [(-x)^3 - 1]^2 = (-x^3 - 1)^2 = (x^3 + 1)^2 \neq P(x)$
$\neq -P(x)$

$\therefore$ neither

5. Explain why the following polynomial function does not have any real zeros. [C 2]

$P(x) = x^4 + x^2 + 2$

$x^4 \geq 0$

$x^2 \geq 0$

$P(x) \geq 2$ for all $x \in \mathbb{R}$

$\therefore$ $P(x)$ does not have any real zeros.

All zeros (4) are complex numbers.

6. Factor completely each polynomial function given below. [A 4]

a) $P(x) = (1 - x^3)(x^2 - 1)$

$P(x) = -(x^3 - 1)(x^2 - 1)$

$= -(x-1)(x^2 + x + 1)(x-1)(x+1)$

$\therefore P(x) = -(x-1)^2(x+1)(x^2 + x + 1)$

b) $P(x) = x^6 - 64$

$P(x) = (x^3)^2 - 8^2 = (x^3 - 8)(x^3 + 8)$

$\therefore P(x) = (x-2)(x^2 + 2x + 4)(x+2)(x^2 - 2x + 4)$

7. Find the parameter a such that the polynomial function $P(x) = x^3 - ax^2 + 2x + 3a$ has $x + 2$ as a factor. [A 3]

$P(-2) = 0$

$0 = -8 - 4a - 4 + 3a$

$\therefore a = -12$

$\overline{11}$

8. Solve the following polynomial inequality. Show your work. [A 3]

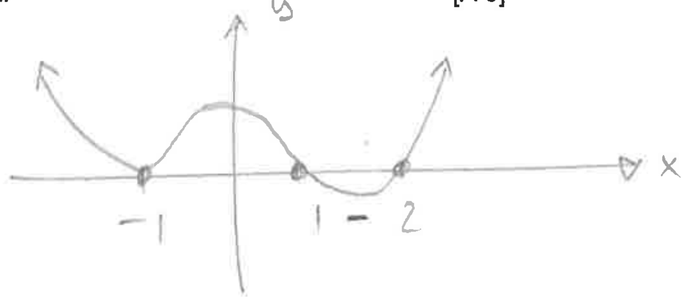
$(x^2-1)(x^2-x-2)<0$

$(x-1)(x+1)(x-2)(x+1) < 0$

$(x-1)(x+1)^2(x-2) < 0$

$\therefore\ x \in (+1, 2)$

or $1 < x < 2$



9. Factor completely. Show your work. [A 3]

$P(x)=x^3-3x+2$

Any integral zero $\in \{\pm 1, \pm 2\}$

$P(1)=0$

$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & 0 & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

x^2+x-2

$(x+2)(x-1)$

$\therefore\ P(x)=(x-1)^2(x+2)$

10. Find the equation the polynomial function given below graphically. Show your work. [A 4]

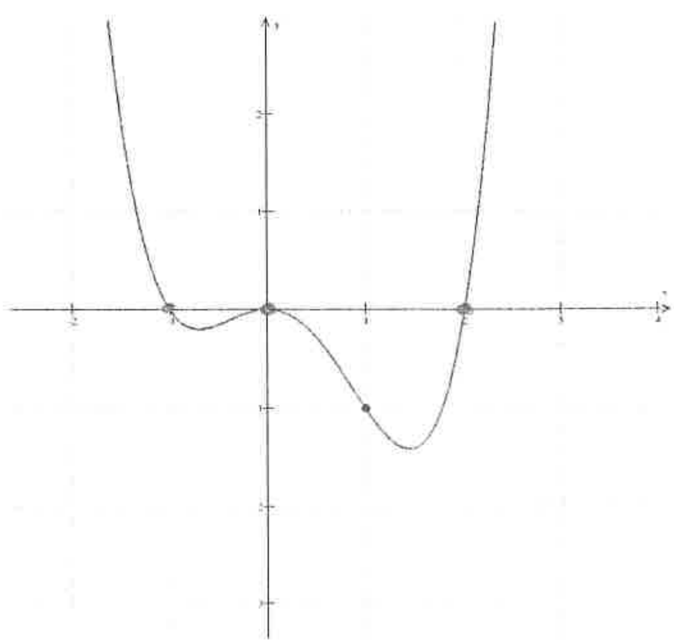
$P(x)=ax^2(x-2)(x+1)$

$P(1)=-1$

$-1=a(-1)(2)$

$a=\frac{1}{2}$

$\therefore\ P(x)=\frac{1}{2}x^2(x-2)(x+1)$



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11. Show that x^2-2 is a factor of $P(x)=x^4-3x^2+2$. [A 3]

$$\begin{array}{r} x^2-1 \\ x^2-2\,\overline{)\,x^4-3x^2+2} \\ x^4-2x^2 \\ \hline -x^2+2 \\ -x^2+2 \\ \hline 0 \end{array}$$

$P(x) = (x^2-2)(x^2-1)$

$\therefore$ $r(x)=0$

x^2-2 is a factor of $P(x)$

12. Show that the following table of values may be modelled by a polynomial function. What is the degree of this polynomial? [T 3]

x	y	Δ^1	Δ^2	Δ^3	Δ^4
-5	1050				
-4	480	-570	270		
-3	180	-300	168	-102	24
-2	48	-132	90	-78	24
-1	6	-42	36	-54	24
0	0	-6	6	-30	24
1	0	0	0	-6	24
2	0	0	18	18	24
3	18	18	60	42	24
4	96	78	126	66	24
5	300	204			

$\therefore$ Δ^4 are constant

$\therefore$ $n=4$

13. Solve the following polynomial inequality. [T 3]

$x^3 \le 2(3x+2)$

$P(x) = x^3 - 6x - 4 \le 0$

Any integral zeros $\in \{\pm 1, \pm 2, \pm 4\}$

$P(1) = -9 \neq 0$

$P(-1) = -1+6-4 = 1 \neq 0$

$P(2) = 8-12-4 = -8 \neq 0$

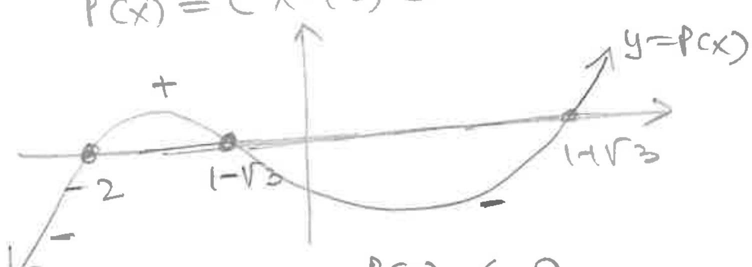
$P(-2) = -8+12-4 = 0$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & -6 & -4 \\ & & -2 & 4 & 4 \\ \hline & 1 & -2 & -2 & 0 \end{array}$$

$x^2-2x-2=0$

$x = \dfrac{2 \pm \sqrt{4+8}}{2} = \dfrac{2 \pm 2\sqrt{3}}{2} = 1 \pm \sqrt{3}$

$P(x) = (x+2)(x-1-\sqrt{3})(x-1+\sqrt{3})$



$P(x) \le 0$

$\therefore$ $x \in (-\infty, -2] \cup [1-\sqrt{3}, 1+\sqrt{3}]$

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