Questions 1-5 are Multiple-Choice questions

1. The equation(s) of the vertical asymptotes for \( f(x) = \frac{x + 2}{x^2 - 1} \) is (are):
   A) \( x = 1 \)  B) \( x = -1 \)  C) \( x = -2 \)  D) \( x = -1 \) or \( x = 1 \)  E) \( x = 2 \)

2. The zero(s) of the rational function \( f(x) = \frac{(x-1)(x+2)}{x-3} \) is (are):
   A) \( x = 1 \)  B) \( x = -1 \)  C) \( x = 1 \) or \( x = -2 \)  D) \( x = -1 \) or \( x = 2 \)  E) \( x = 3 \)

3. For the polynomial function \( P(x) = x^3 - 3x^2 + kx - 6 \) (where \( k \) is integer) which of the following integers is NOT a possible integral zero:
   A) 1  B) 2  C) 3  D) 4  E) 6

4. The equation of the horizontal asymptote of the polynomial function \( f(x) = \frac{4x^3 - 3x + 2}{1 - 2x^3} \) is:
   A) \( y = 2 \)  B) \( y = 4 \)  C) \( y = -2 \)  D) \( y = -3 \)  E) \( y = 0 \)

5. The coordinates of the hole for the function \( f(x) = \frac{x^2 - 1}{x + 1} \) are:
   A) \((-1,-1)\)  B) \((1,2)\)  C) \((-1,-2)\)  D) \((-1,2)\)  E) \((-1,-2)\)

Questions 6-10 are True-False questions

6. An oblique asymptote cannot be intersected by the graph of a rational function.
   T  F

7. A hole appears in the graph of a rational function for any common zero of the numerator and denominator.
   T  F

8. The zeros of a rational function are the zeros of the numerator which are not zeros of the denominator.
   T  F

9. If the degree of the numerator is greater than the degree of the denominator then the \( x \)-axis is the horizontal asymptote of the rational function.
   T  F

10. A polynomial function of degree 3 has at least one real zero.
    T  F

11. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph.

   A) \[ \times \]
   \[ f(x) = \frac{1}{x + 1} \]

   B) \[ \parallel \]
   \[ g(x) = \frac{1}{x - 1} \]

   C) \[ \parallel \]
   \[ h(x) = \frac{1}{x^2 - 1} \]

   D) \[ \parallel \]
   \[ k(x) = \frac{1}{x^2 + 1} \]

   E) \[ \times \]
   \[ p(x) = \frac{1}{(x + 1)^2} \]

   F) \[ \parallel \]
   \[ q(x) = \frac{1}{(x - 1)^2} \]
12. Sketch the graph of the following functions on the grid provided.

\[ f(x) = \frac{-6}{x+2} \]

\[ g(x) = \frac{-1}{x^2 + x} = \frac{-1}{x(x+1)} \]

\[ h(x) = \frac{2}{(x-2)^2} \]

\[ h(x) = \frac{3}{x^2 + 1} \]

\[ h(x) = \frac{2x-3}{1-x} \]

\[ h(x) = \frac{x^2 - 4}{x-2} = \frac{x+2}{x} \quad \text{for} \quad x \neq 2 \]

\[ y - iu = 2 \]

\[ x - (i) = -2 \]
13. Sketch the graph of the polynomial function \( f(x) = -2x^2(x-1)(x+2)^2(x-3)^3 \). [A 3 marks]

14. Factorize completely \( P(x) = x^4 + 3x^3 - 7x^2 - 27x - 18 \).

\[
\begin{align*}
\text{Integral Zero: } & \pm (1, 2, 3, 6, 9, 18) \\
P(1) &= 1 + 3 - 7 - 27 - 18 = 0 \\
P(-1) &= 1 - 3 - 7 + 27 - 18 = 0 \\
P(2) &= 16 + 6 - 28 - 54 - 18 = 0 \\
P(-2) &= 0 - 8 - 28 - 54 - 18 = 0 \\
P(3) &= 81 + 27 - 63 - 81 - 18 = 0 \\
P(-3) &= 0 - 27 + 18 - 81 - 18 = 0 \\
P(x) &= (x-1)(x+2)(x-3)(x+3) \\
P(C) &= x^2 - 9 \\
&= (x-3)(x+3)
\end{align*}
\]

15. Determine the equation of a quartic function (degree is four) having the zeros \( \pm 1 \) and \( \pm 2 \) and the y-intercept point \( P(0, 4) \). [A 3 marks]

\[
P(x) = (x-1)(x+1)(x-2)(x+2) \\
P(0, 4) \\
x = 0 \\
y = 4 \\
4 = \alpha \cdot (-1) \cdot (-1) \\
\alpha = 1
\]

16. Solve the inequality \(-2x^3 - 6x^2 + 12x + 16 \leq 0\). [A 4 marks]

\[
P(x) = -2x^3 - 6x^2 + 12x + 16
\]

Integral Zero: \( \pm (1, 2, 3, 6, 8, 16) \\
P(1) = -2 - 6 + 12 + 16 \neq 0 \\
P(-1) = 2 - 6 - 12 + 16 = 0 \\
P(2) = 32 - 24 + 24 + 16 = 40 \\
P(-2) = -32 - 24 - 24 + 16 = -40
\]

\[
\text{Solution: } [-4, -2] \cup [2, +\infty]
\]

17. Use the long division algorithm to find the quotient and the remainder for the following division of two polynomials. [A 3 marks]

\[
\begin{array}{l}
2x^4 - 3x^3 + x - 1 \\
x^3 - 2x + 1
\end{array}
\]

\[
\begin{array}{c}
\underline{2x^4 + 4x^3 + 3} \\
2x^4 - 3x^3 + x - 1 \\
\underline{4x^3 + 5x^2 + x} \\
4x^3 - 8x^2 + 4x \\
\underline{3x^2 - 3x - 1} \\
3x^2 - 6x + 3 \\
\underline{3x - 4}
\end{array}
\]

\[
\begin{align*}
\text{Quotient: } & Q(x) = 2x^2 + 4x + 3 \\
\text{Remainder: } & R(x) = 3x - 4
\end{align*}
\]
18. Factor completely \( P(x) = 12x^3 + 8x^2 - 3x - 2 \).

Rational Zeros \( \pm \frac{1}{b} \) \( \pm \frac{a}{c} \) \( \pm \frac{b}{a} \) \( \pm \frac{c}{a} \) \( \pm \frac{d}{a} \)

\[
P(1) = 12 + 8 - 3 - 2 = 0
\]

\[
P(-1) = -12 + 8 + 3 - 2 = 0
\]

\[
P(2) = 36 + 32 - 6 - 2 = 0
\]

\[
P(-2) = -48 + 32 + 6 - 2 = 0
\]

\[
P\left( \frac{1}{2} \right) = 12 + \frac{1}{8} + \frac{1}{4} - \frac{3}{2} - 2
\]

\[
= \frac{32}{8} + \frac{2}{8} - \frac{12}{8} - 2 = 0
\]

\[
\begin{array}{c|cccc}
 & 12 & 8 & -3 & -2 \\
\hline
12 & 14 & 4 & 0 \\
\hline
\end{array}
\]

\[
\theta(x) = 12x^2 + 14x + 4
\]

\[
= 2\left(6x^2 + 7x + 2\right)
\]

19. Sketch the graph of \( f(x) = \frac{1+\frac{1}{x}}{1+\frac{1}{x} - \frac{3}{x}} \) \( x \neq 0 \), \( x \geq 3 \).

\[
f(x) = \frac{x+1}{x} \cdot \frac{x}{x-3 + 2x}
\]

\[
= \frac{x+1}{x} \cdot \frac{x}{x(x-3)}
\]

\[
= \frac{x+1}{x} \cdot \frac{x(x-3)}{3(x-1)}
\]

\[
d(x) = \frac{(x+1)(x-3)}{3(x-1)} \quad ; \quad x \neq 0, 3 \geq 1
\]

\( V_A \) \( V_H \) \( O_A \) \( \frac{x-1}{x-3} \)

Holes: \( (0,1) \) and \( (3,6) \)