

MHF4U

Review: Polynomial Functions

1. Classify each polynomial function as even, odd, or neither.

a) $f(x) = 2x^5 - 3x^2 + 1$ neither

b) $f(x) = -x^4 + 5x^2 - 4$ even

c) $f(x) = -x^3 + 2x$ odd

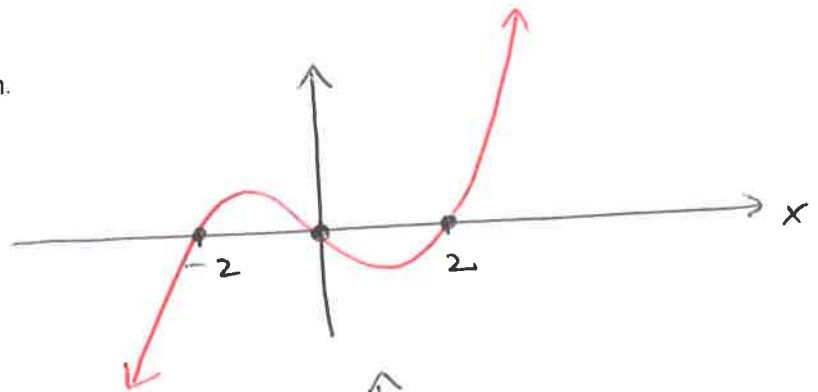
2. Analyze the end behavior of each polynomial function.

a) $f(x) = 1 - 2x^4 + x^3 - x$ As $x \rightarrow \pm \infty, y \rightarrow -\infty$

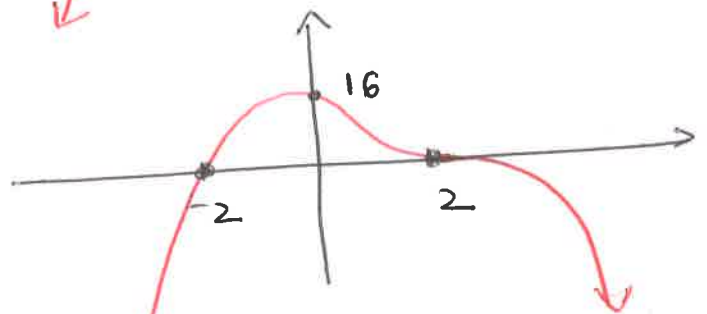
b) $f(x) = -2x^2 + 3x^5 - x^3$ As $x \rightarrow \infty, y \rightarrow \infty$
As $x \rightarrow -\infty, y \rightarrow -\infty$

3. Sketch the graph of each polynomial function.

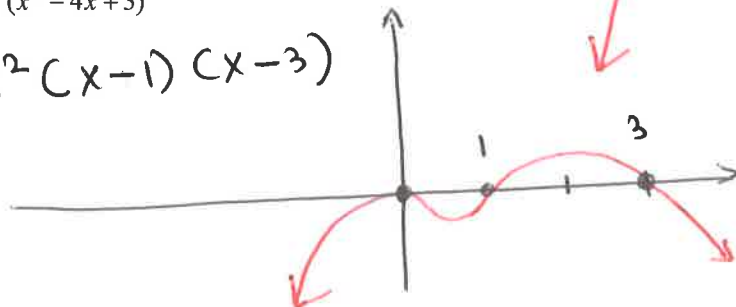
a) $f(x) = x^3 - 4x$
 $= x(x^2 - 4)$
 $= x(x-2)(x+2)$



b) $f(x) = (x-2)^2(4-x^2)$
 $= (x-2)^2(2-x)(2+x)$
 $= -(x+2)(x-2)^3$

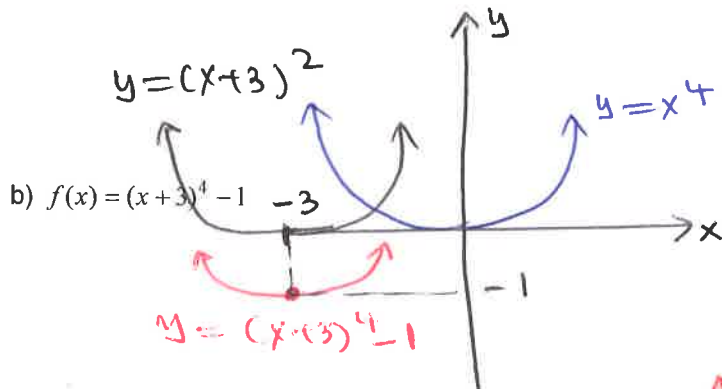


c) $f(x) = -2x^2(x^2 - 4x + 3)$
 $= -2x^2(x-1)(x-3)$



4. Use transformations to graph each polynomial function.

a) $f(x) = -(x-2)^3 + 4$

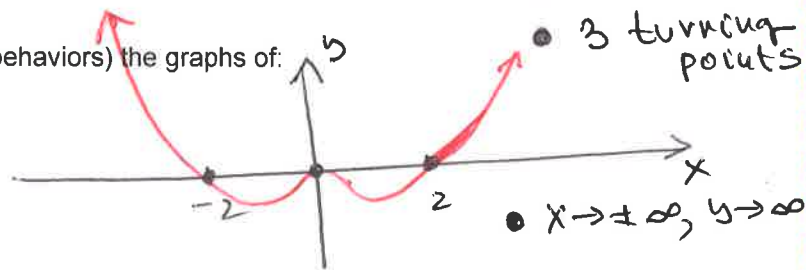


5. Compare (x-intercepts, turning points, and end behaviors) the graphs of:

a) $f(x) = x^4 - 4x^2$

$$= x^2(x^2 - 4)$$

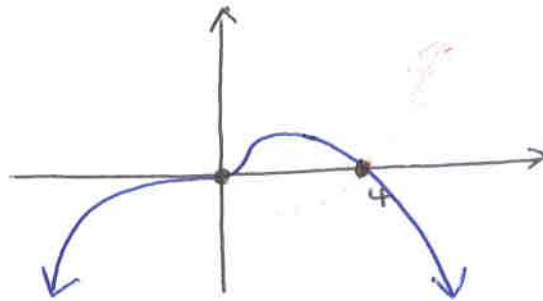
$$= x^2(x-2)(x+2)$$



b) $f(x) = 4x^3 - x^4$

$$= x^3(4-x)$$

$$= -x^3(x-4)$$



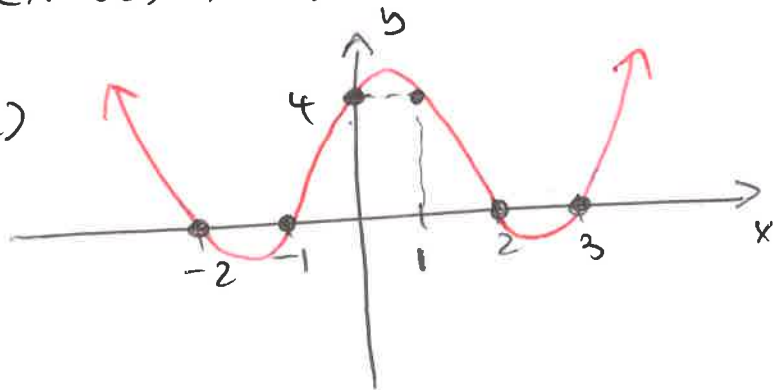
7. Find a polynomial function of degree four with the x-intercepts $-1, 2, -2, 3$ whose graph passes through the point $P(1,4)$. Sketch the graph.

$$f(x) = a(x+1)(x-2)(x+2)(x-3)$$

$$4 = f(1)$$

$$4 = a(2)(-1)(3)(-2)$$

$$a = \frac{4}{12} = \frac{1}{3}$$



$$\therefore f(x) = \frac{1}{3}(x+1)(x-2)(x+2)(x-3)$$

$$y\text{-cut} = 4$$

Consider the following polynomial function:

$$f(x) = x(x^2 + 3x - 4) = x^3 - 3x^2 - 4x$$

a) Classify this function as even, odd or neither. /2

$$f(-x) = -x^3 - 3x^2 + 4x$$

$$\neq f(x)$$

$$\neq -f(x)$$

! Some exponents of x are odd,
and some are even.

$\therefore f$ is neither, even nor odd

b) Analyze the end behavior. /2

$$a_n = 1 > 0$$

$$f(x) \approx x^3 \text{ as } x \rightarrow \pm\infty$$

$$\text{As } x \rightarrow \infty, y \rightarrow \infty$$

$$\text{As } x \rightarrow -\infty, y \rightarrow -\infty$$

c) Find the x-intercepts and the y-intercept. /3

$$y\text{-int} = f(0) = 0$$

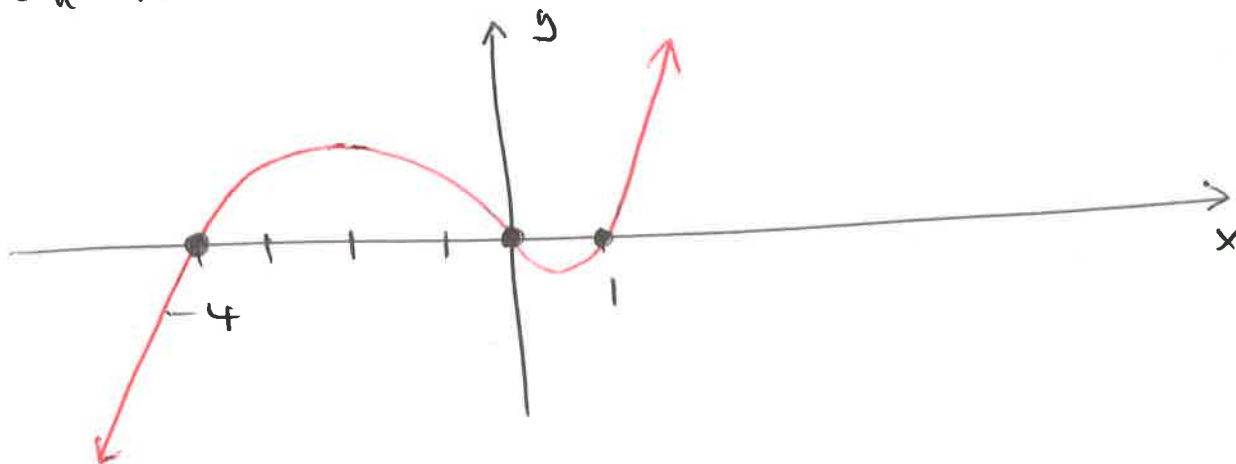
$$f(x) = x(x+4)(x-1)$$

x-int are $0, -4, \text{ and } 1$

d) Sketch the graph of this function. /3

Simple zeros: $0, -4, 1$

$$a_n = 1 > 0$$



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Quiz Polynomial Functions

Name SOLUTIONS
Date Aug 24, 2015

Consider the following polynomial function:

$$f(x) = x(x^2 - x - 6) = x^3 - x^2 - 6x$$

- a) Classify this function as even, odd or neither.
- /2

$$\begin{aligned} f(-x) &= -x^3 - x^2 + 6x \\ &\neq f(x) \\ &\neq -f(x) \end{aligned}$$

Ⓢ Some exponents of x are odd, some are even.

∴ f is neither odd nor even

- b) Analyze the end behavior.
- /2

$$a_n = 1$$

$$f(x) \approx x^3 \text{ as } x \rightarrow \pm \infty$$

$$\text{As } x \rightarrow \infty, y \rightarrow \infty$$

$$\text{As } x \rightarrow -\infty, y \rightarrow -\infty$$

- c) Find the x-intercepts and the y-intercept.
- /3

$$y\text{-cut} = f(0) = 0$$

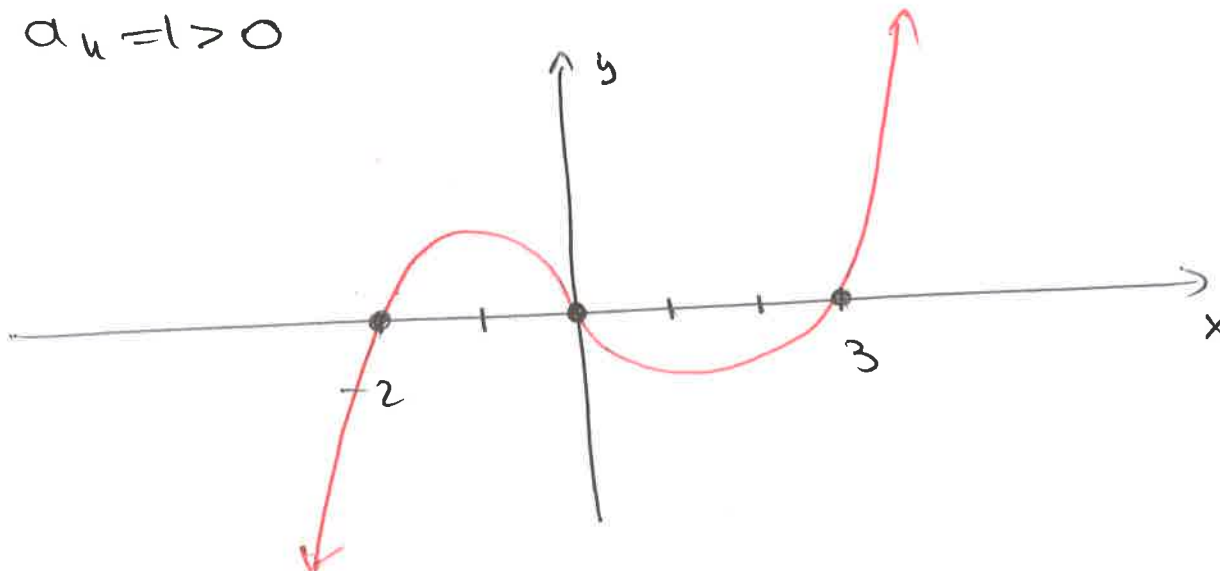
$$f(x) = x(x-3)(x+2)$$

$$x\text{-cut are } 0, 3, \text{ and } -2$$

- d) Sketch the graph of this function.
- /3

Simple zeros: $0, 3, -2$

$$a_n = 1 > 0$$



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Quiz Polynomial Functions

Name SOLUTIONS

Date Aug 22, 2016

Consider the following polynomial function:

$$f(x) = x(x^2 - 4)(x^2 - 1)$$

- a) Classify this function as even, odd or neither. Explain.

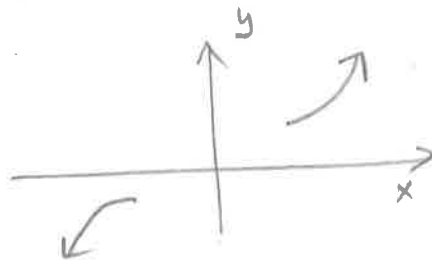
/2

$$\begin{aligned} f(-x) &= -x [(-x)^2 - 4] [(-x)^2 - 1] \\ &= -x (x^2 - 4)(x^2 - 1) \\ &= -f(x) \\ \therefore f \text{ is odd} \end{aligned}$$

- b) Analyze the end behavior.

/2

$$\begin{aligned} f(x) &= x(x^4 - 5x^2 + 4) \\ &= x^5 - 5x^3 + 4x \\ \text{LT} &= x^5 \\ \text{LC} &= 1 > 0 \end{aligned}$$



As $x \rightarrow \pm \infty$, $y \rightarrow \pm \infty$

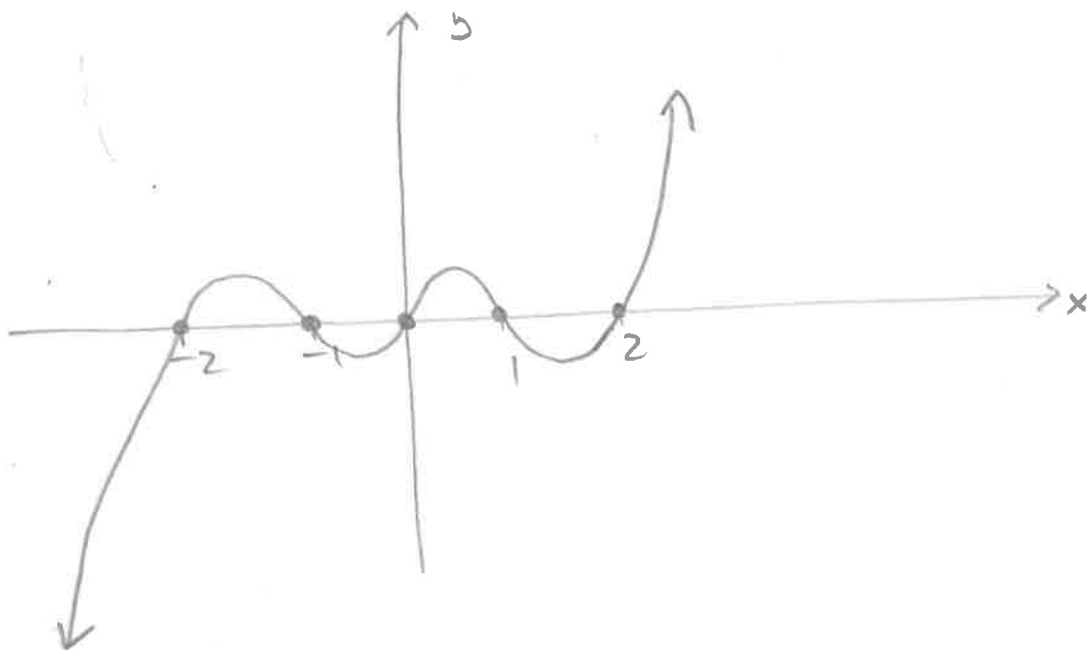
- c) Find the x-intercepts and the y-intercept.

/3

$$\begin{aligned} f(x) &= x(x-2)(x+2)(x-1)(x+1) \\ x\text{-int} &= 0, \pm 2, \pm 1 \quad (\text{simple zeros}) \\ y\text{-int} &= 0 \end{aligned}$$

- d) Sketch the graph of this function.

/3



Consider the following polynomial function:

$$f(x) = (9 - x^2)(x^2 - 1)$$

- a) Classify this function as even, odd or neither. Explain.

/2

$$\begin{aligned} f(-x) &= [9 - (-x)^2][(-x)^2 - 1] \\ &= (9 - x^2)(x^2 - 1) \\ &= f(x) \\ \therefore f &\text{ is even} \end{aligned}$$

- b) Analyze the end behavior.

/2

$$\begin{aligned} f(x) &= -x^4 + 10x^2 - 9 \\ \text{Leading Term} &= -x^4 \\ \text{Leading Coefficient} &= -1 < 0 \end{aligned}$$



$$\therefore \text{As } x \rightarrow \pm \infty, y \rightarrow -\infty$$

- c) Find the x-intercepts and the y-intercept.

/3

$$\begin{aligned} f(x) &= (3-x)(3+x)(x-1)(x+1) \\ &= -(x-3)(x+3)(x-1)(x+1) \end{aligned}$$

$$\therefore \text{x-int} = 3, -3, 1, -1 \text{ (simple zeros)}$$

$$\therefore \text{y-int} = f(0) = -9$$

- d) Sketch the graph of this function.

/3

