

Note. Show your work to get full marks.

1. Given is the polynomial function $f(x) = 2x^2(9 - x^2)$.

a) Expand and find the leading and the constant term.

[K 2]

$$f(x) = -2x^4 + 18x^2$$
$$LT = -2x^4$$
$$CT = 0$$

b) Find if the polynomial function is even, odd, or neither.

[K 2]

Exponents : 4 and 2 (all even)
f is even

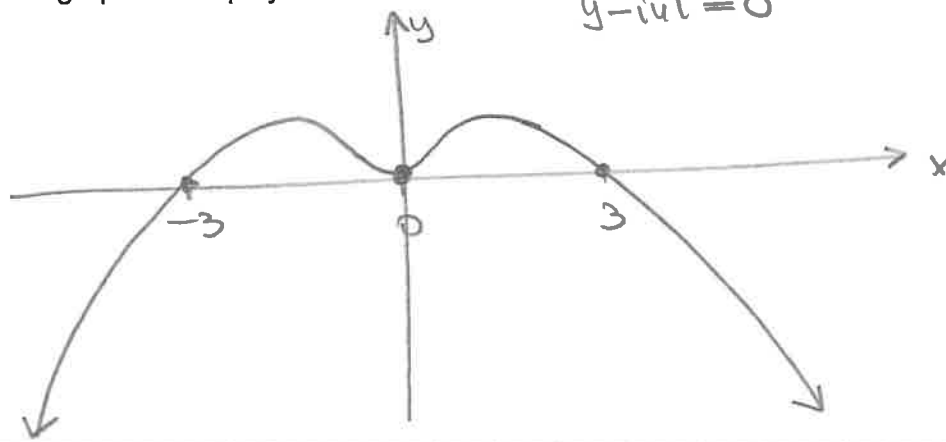
c) Factorize and find its zeros and their multiplicities (orders).

[K 2]

$$f(x) = -2x^2(x-3)(x+3)$$
$$x_1 = 0 ; m_1 = 2$$
$$x_2 = 3 ; m_2 = 1$$
$$x_3 = -3 ; m_3 = 1$$

d) Sketch the graph of this polynomial function.

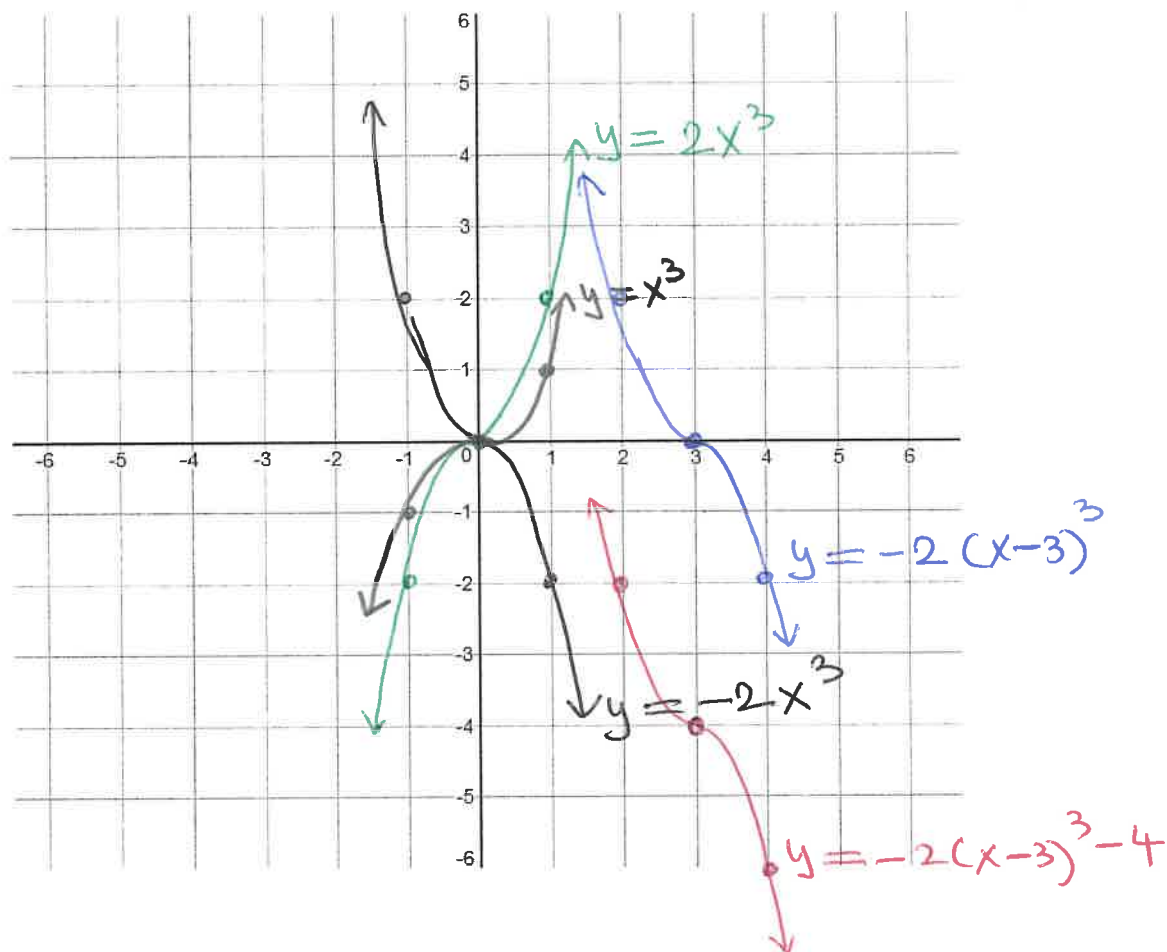
[T 2]



2. Use transformations to graph $f(x) = 2(3-x)^3 - 4$

[A 4]

$$= -2(x-3)^3 - 4$$



3. Classify the following relation as linear, quadratic, cubic or quartic. Explain.

[C 3]

X	Y	Δ_1	Δ_2	Δ_3	Δ_4
-3	-138				
-2	-26	112			
-1	-2	24	-88		
0	0	2	-22	66	-48
1	-2	-2	-4	18	-48
2	-38	-36	-34	-30	-48
3	-186	-148	-112	-78	

Fourth order differences are constant

$\therefore n = 4$
(Quartic)

Note. Show your work to get full marks.

1. Given is the polynomial function $f(x) = -x(x^2 - 1)^2$.

a) Expand and find the leading and the constant term. [K 2]

$$\begin{aligned} f(x) &= -x(x^4 - 2x^2 + 1) \\ f(x) &= -x^5 + 2x^3 - x \\ \text{LT} &= -x^5 \\ \text{CT} &= 0 \end{aligned}$$

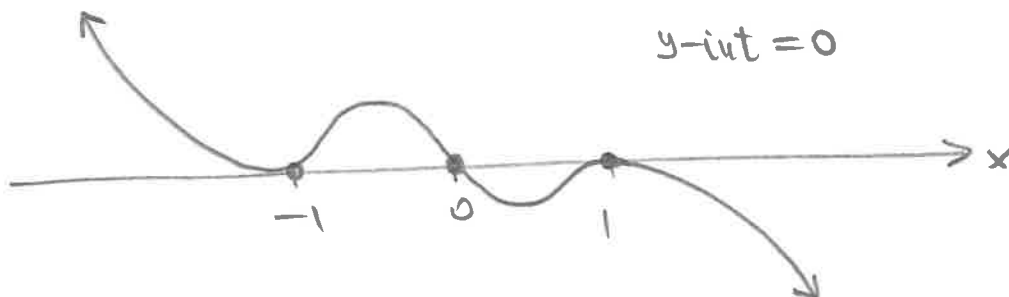
b) Find if the polynomial function is even, odd, or neither. [K 2]

Exponents of x : 5, 3, 1 (all odd)
 f is odd

c) Factorize and find its zeros and their multiplicities (orders). [K 2]

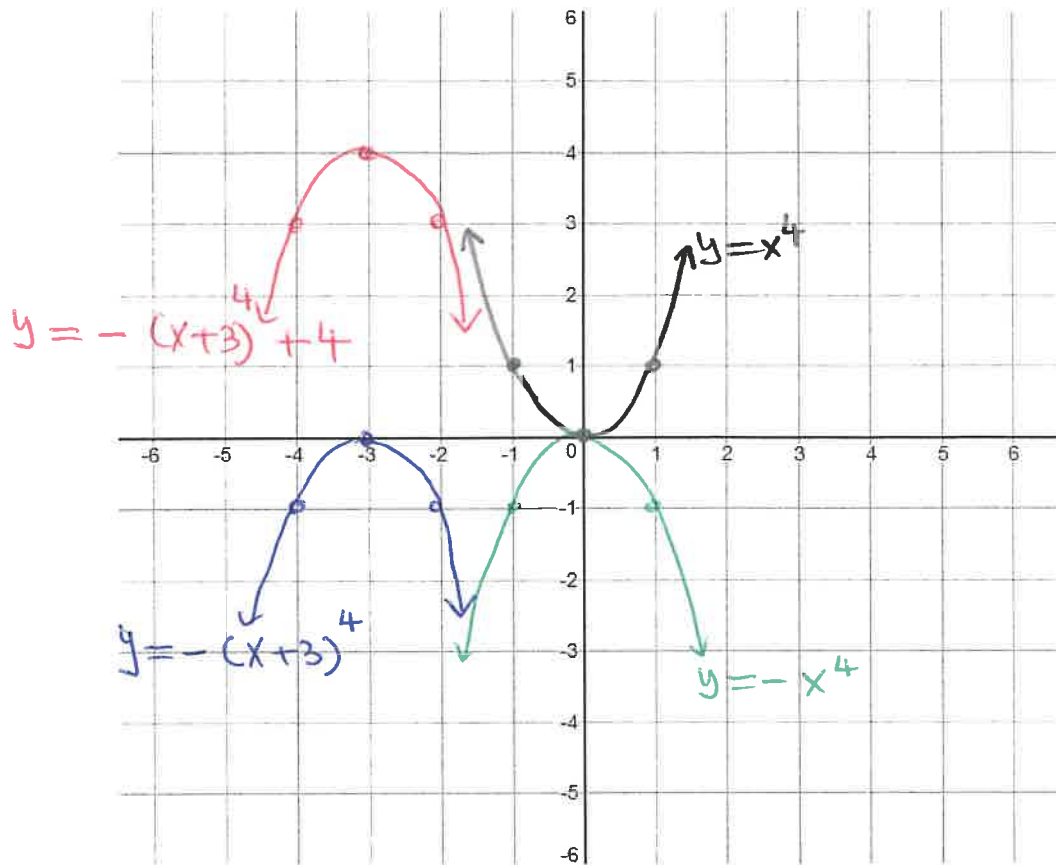
$$\begin{aligned} f(x) &= -x(x-1)^2(x+1)^2 \\ x_1 &= 0 ; m_1 = 1 \\ x_2 &= 1 ; m_2 = 2 \\ x_3 &= -1 ; m_3 = 2 \end{aligned}$$

d) Sketch the graph of this polynomial function. [T 2]



2. Use transformations to graph $f(x) = -(x+3)^4 + 4$

[A 4]



3. Classify the following relation as linear, quadratic, cubic or quartic. Explain.

[C 3]

x	y	Δ_1	Δ_2	Δ_3
-3	55	-35		
-2	20	-17	18	
-1	3	-5	12	-6
0	-2	1	6	-6
1	-1	1	0	-6
2	0	-5	-6	-6
3	-5			

Third order differences are constant

$$\therefore n = 3$$

Relation is cubic

Note. Show your work to get full marks.

$$4x^2 - 2x^3 - 16 + 8x$$

1. Given is the polynomial function $f(x) = (x^2 - 4)(4 - 2x)$.

a) Expand and find the leading and the constant term. [K 2]

$$f(x) = -2x^3 + 4x^2 + 8x - 16$$

$$LT = -2x^3$$

$$CT = -16$$

b) Find if the polynomial function is even, odd, or neither. [K 2]

f is neither odd nor even
(even and odd exponents of x)

c) Factorize and find its zeros and their multiplicities (orders). [K 2]

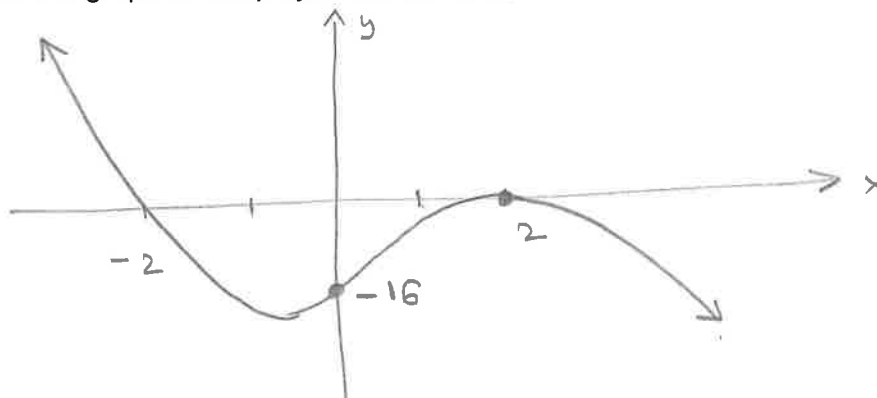
$$f(x) = (x-2)(x+2)(2)(2-x)$$

$$= -2(x-2)^2(x+2)$$

$$x_1 = 2 ; m_1 = 2$$

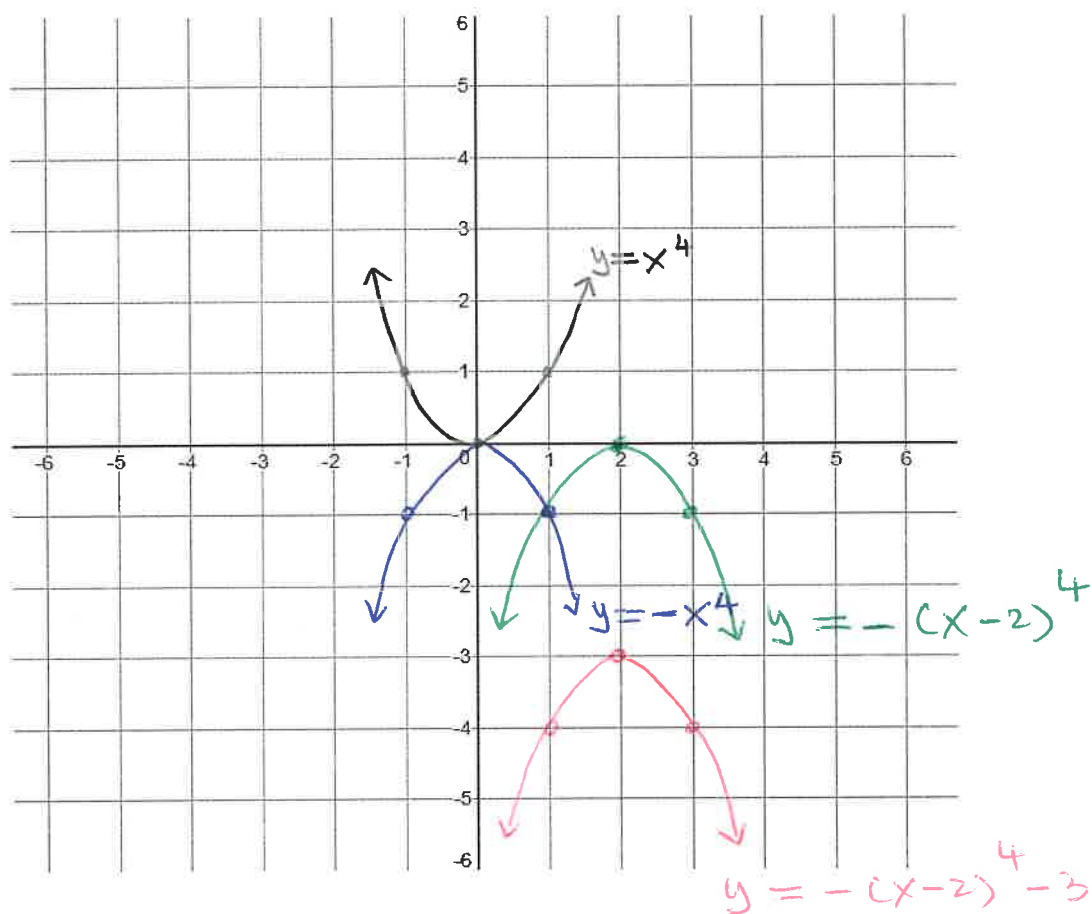
$$x_2 = -2 ; m_2 = 1$$

d) Sketch the graph of this polynomial function. [T 2]



2. Use transformations to graph $f(x) = -(x-2)^4 - 3$

[A 4]



3. Classify the following relation as linear, quadratic, cubic or quartic. Explain.

[C 3]

x	y	Δ^1	Δ^2	Δ^3
-3	60	> -42	> 26	> -12
-2	18	> -16	> 14	> -12
-1	2	> -2	> 2	> -12
0	0	> 0	> -10	> -12
1	0	> -10	> -22	> -12
2	-10	> -32		
3	-42			

$n = 3$
cubic

constant

Note. Show your work to get full marks.

1. Given is the polynomial function $f(x) = (x+1)^2(1-x^2) = (x^2+2x+1)(1-x^2)$

a) Expand and find the leading and the constant term. [K 2]

$$\begin{aligned} f(x) &= x^2 + 2x + 1 - x^4 - 2x^3 - x^2 \\ &= -x^4 - 2x^3 + 2x + 1 \\ \text{LT} &= -x^4 \\ \text{CT} &= 1 \end{aligned}$$

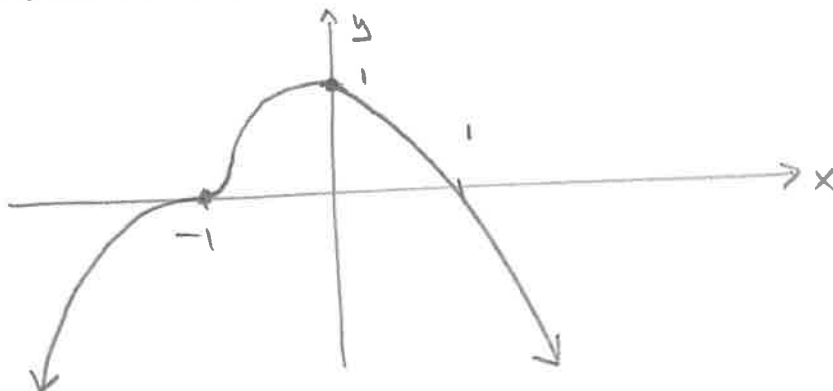
b) Find if the polynomial function is even, odd, or neither. [K 2]

neither
(odd and even exponents of x)

c) Factorize and find its zeros and their multiplicities (orders). [K 2]

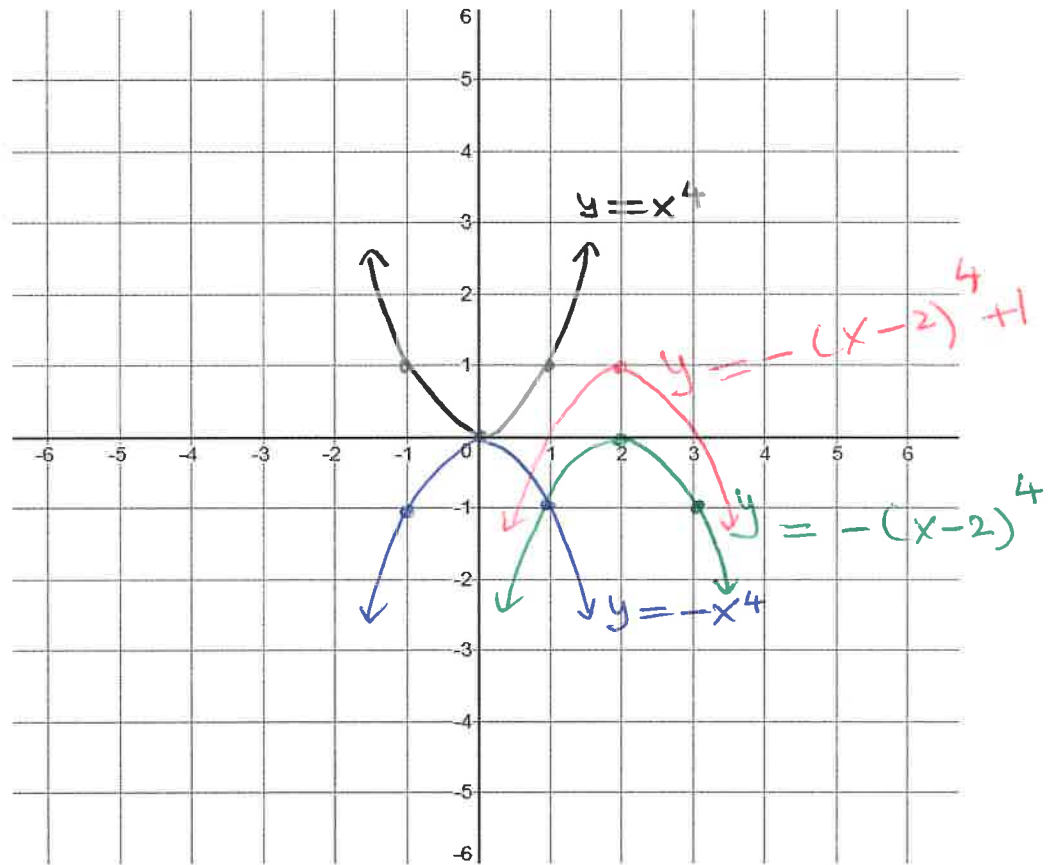
$$\begin{aligned} f(x) &= (x+1)^2(1-x)(1+x) \\ &= -(x-1)(x+1)^3 \\ x_1 &= 1 ; w_1 = 1 \\ x_2 &= -1 ; w_2 = 3 \end{aligned}$$

d) Sketch the graph of this polynomial function. [T 2]



2. Use transformations to graph $f(x) = -(x-2)^4 + 1$

[A 4]



3. Classify the following relation as linear, quadratic, cubic or quartic. Explain.

[C 3]

x	y	Δ^1	Δ^2	Δ^3	Δ^4
-3	-120	> 90	> -64	> 42	> -24
-2	-30	> 26	> -22	> 18	> -24
-1	-4	> 4	> -4	> -6	> -24
0	0	> 0	> -10	> -30	> -24
1	0	> -10	> -40		
2	-10	> -50			
3	-60				

$\therefore n = 4$
 .. quartic

↑
 constant

Assignment: Polynomial Functions

1. Find the quotient and the remainder. Show your work.

[K/U 4]

a) $\frac{2x^5 - 3x^2 + 1}{x^3 - x - 2}$

$$\begin{array}{r}
 2x^2 + 2 \\
 x^3 - x - 2 \overline{) 2x^5 - 3x^2 + 1} \\
 \underline{2x^5 - 2x^3 - 4x^2} \\
 2x^3 + x^2 + 1 \\
 \underline{2x^3 - 2x - 4} \\
 x^2 + 2x + 5
 \end{array}$$

$$\therefore Q(x) = 2x^2 + 2$$

$$R(x) = x^2 + 2x + 5$$

2. Show that $x+2$ is not a factor of the polynomial $P(x) = 2x^4 + x^3 - x + 1$ by using two different methods. Explain.

[C 4]

By using
Synthetic Division

$$\begin{array}{r|rrrrr}
 -2 & 2 & 1 & 0 & -1 & 1 \\
 & & -4 & 6 & -12 & 26 \\
 \hline
 & 2 & -3 & 6 & -13 & 27
 \end{array}$$

$$r = 27 \neq 0$$

 $\therefore x+2$ is not a factor
of $P(x)$ By using
Remainder Theorem

$$\begin{aligned}
 r &= P(-2) \\
 &= 2(-2)^4 + (-2)^3 - (-2) + 1 \\
 &= 32 - 8 + 2 + 1 \\
 &= 27 \neq 0
 \end{aligned}$$

 $\therefore -2$ is not a zero
 $x+2$ is not a factor
of $P(x)$

3. Factor completely. Show your work.

[A 4]

$$P(x) = x^4 - 2x^3 - 3x^2 + 6x$$

$$= x(x^3 - 2x^2 - 3x + 6)$$

Set of factors of 6 is

$$\{\pm 1, \pm 2, \pm 3, \pm 6\}$$

$$f(x) = x^3 - 2x^2 - 3x + 6$$

$$f(2) = 0$$

$$\begin{array}{r|rrrr} 2 & 1 & -2 & -3 & 6 \\ & 0 & 2 & 0 & -6 \\ \hline & 1 & 0 & -3 & 0 \end{array}$$

$$P(x) = x(x-2)(x^2-3)$$

$$\therefore P(x) = x(x-2)(x-\sqrt{3})(x+\sqrt{3})$$

4. Graph the polynomial function. Find all the local and global extrema.

[T 4]

$$P(x) = x^4 - x^2 + \frac{1}{4}$$

$$P(x) = \left(x^2 - \frac{1}{2}\right)^2$$

$$= \left(x - \frac{1}{\sqrt{2}}\right)^2 \left(x + \frac{1}{\sqrt{2}}\right)^2$$

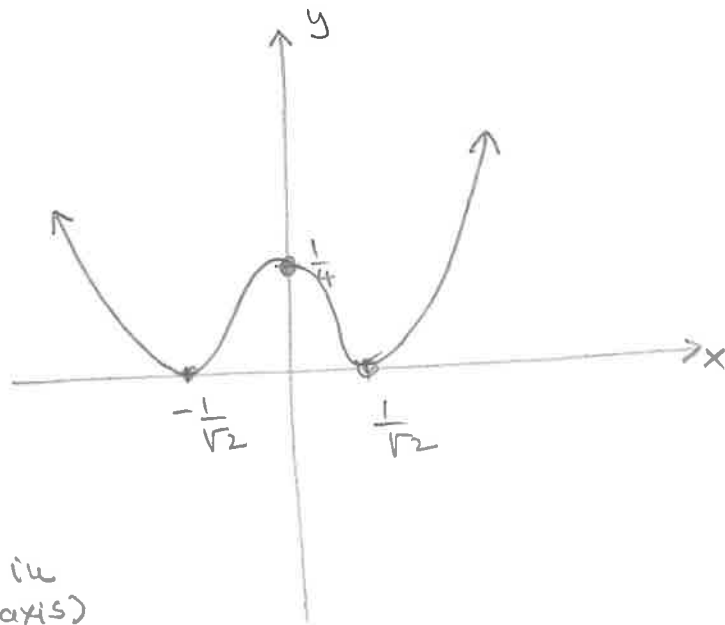
Zeros:

$$x_1 = \frac{1}{\sqrt{2}}; m_1 = 2$$

$$x_2 = -\frac{1}{\sqrt{2}}; m_2 = 2$$

$P(x)$ is even (symmetry in y-axis)

$$P(0) = \frac{1}{4}$$



\therefore Local maximum at $(0, \frac{1}{4})$ \therefore Global maximum: none

\therefore Local minima at $(\pm \frac{1}{\sqrt{2}}, 0)$ \therefore Global minima at $(\pm \frac{1}{\sqrt{2}}, 0)$