3.7 Factoring a Sum or Difference of Cubes (Powers)

A Difference of Two Cubes

The following formula is called the *difference of cubes* identity.

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

$$a^2-b^2=(a-b)(a+b)$$

Ex 1. Use the difference of cubes identity to factor the following polynomial functions:

a) $x^3-8 = x^3-2^3=(x-2)(x^2+2x+4)$

$\Delta = 2^2-4(1)(4)<0$

b^2-4ac

b) $27x^3-64$

$= (3x)^3-4^3 = (3x-4)(9x^2+12x+16)$

c) $\frac{x^3}{27}-125$

$=\left(\frac{x}{3}-5\right)\left(\frac{x^2}{9}+\frac{5x}{3}+25\right)$

B Sum of Two Cubes

The following formula is called the *sum of cubes* identity.

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$a^2+b^2=?$

$a^3-\cancel{a^2b}+\cancel{ab^2}$
$\cancel{ba^2}-\cancel{ab^2}+b^3$

Ex 2. Use the sum of cubes identity to factor the following polynomial functions:

a) $x^3+1=(x+1)(x^2-x+1)$

b) $8x^3+27 = (2x)^3+3^3$

$= (2x+3)(4x^2-6x+9)$

c) $\frac{x^3}{64}+\frac{8}{27} = \left(\frac{x}{4}\right)^3+\left(\frac{2}{3}\right)^3$

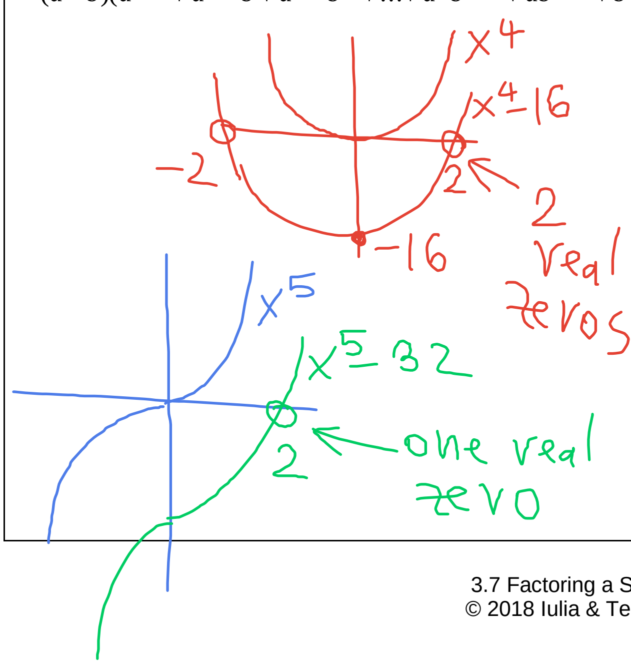
$=\left(\frac{x}{4}+\frac{2}{3}\right)\left(\frac{x^2}{16}-\frac{x}{6}+\frac{4}{9}\right)$

C Difference of Two Powers

For any natural number n, the following identity is true:

$a^n-b^n=$

$=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\ldots+a^2b^{n-3}+ab^{n-2}+b^{n-1})$



Ex 3. Factor as much as you can.

a) x^2-4

$=(x-2)(x+2)$

b) $x^4-16 = x^4-2^4=(x-2)(x^3+2x^2+4x+8)$

$=(x^2)^2-4^2=(x^2-4)(x^2+4)$

$=(x-2)(x+2)(x^2+4)$

Ex 4. Use synthetic division to factor x^5-32.

x^5-2^5

$=(x-2)(x^4+2x^3+4x^2+8x+16)$

D Sum of Two Powers

If n is an *odd* natural number, the following identity is true:

$$a^n + b^n =$$
$$= (a+b)(a^{n-1} - a^{n-2}b + a^{n-3}b^2 - \ldots \pm a^2 b^{n-3} \mp ab^{n-2} \pm b^{n-1})$$

Ex 5. Use synthetic division to factor completely.

a) $x^5 + 1$

$= (x+1)(x^4 - x^3 + x^2 - x + 1)$

b) $x^7 + 128$

$= (x+2)(x^6 - 2x^5 + 4x^4 - 8x^3 + 16x^2 - 32x + 64)$

Ex 6. Use different techniques to factor.

a) $x^6 - 1$ $= x^6 - 1^6 = (x-1)(- - - -)$

$= (x^3)^2 - 1^2 = (x^3 - 1)(x^3 + 1)$

$\to - - -$ $\to - - -$

$= (x^2)^3 - 1^3 = (x^2 - 1)(- - - - -)$

b) $x^{10} - 1$

c) $x^9 + 1$

Ex 7. Given that $a - b = 4$ and $ab = 2$, find $a^3 - b^3$.

Reading: Nelson Textbook, Pages 178-181
Homework: Nelson Textbook, Page 182: #2ac, 3ac, 5a, 6, 8

6a

$$x^6-1$$

$$=(x^3)^2-1^2=(x^3-1)(x^3+1)$$

$$=(x-1)(x^2+x+1)(x+1)(x^2-x+1)$$

$$(x^2)^3-1^3=(x^2-1)(x^4+x^2+1)$$

$$=(x-1)(x+1)(x^4+x^2+1)$$

$$x^6-1^6=(x-1)(x^5+x^4+x^3+x^2+x+1)$$

6b

$$x^{10}-1$$

$$(x5)^2-1^2=(x^5-1)(x^5+1)$$

$$=(x-1)(x^4+x^3+x^2+x+1)(x+1)(x^4-x^3+x^2-x+1)$$

$$(x^2)^5-1=(x^2-1)(x^8+x^6+x^4+x^2+1)$$

$$=(x-1)(x+1)(x^8+x^6+x^4+x^2+1)$$

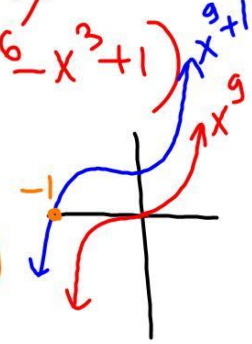
$$(x-1)(x^9+x^8+x^7+x^6+x^5+x^4+x^3$$
$$+x^2+x+1)$$

⑥c x^9+1

$\to (x^3)^3+1^3=(x^3+1)(x^6-x^3+1)$

$=(x+1)(x^2-x+1)(x^6-x^3+1)$

$\to (x+1)(x^8-x^7+x^6-x^5+x^4-x^3+x^2-x+1)$



⑦ $a^3-b^3=(a-b)(a^2+ab+b^2)$

$(a-b)^2=a^2+b^2-2ab$

$=(a-b)\left[(a-b)^2+3ab\right]$

$=(4)\left[4^2+3(2)\right]=88$