### 3.6 Factoring Polynomials

#### A The Remainder Theorem

If a polynomial \( P(x) \) is divided by \( x - b \) then the remainder is \( r = P(b) \).

**Proof:**

| Ex 1. Determine the remainder when \( P(x) = 2x^3 - 4x^2 + 3x - 6 \) is divided by |
|---------------------------------|-------------------|
| a) \( x - 2 \)                 |                   |
| b) \( x + 1 \)                 |                   |

<table>
<thead>
<tr>
<th>Ex 2. When ( P(x) = x^3 - kx^2 + 17x + 6 ) is divided by ( x - 3 ), the remainder is 12. Find the value of ( k ).</th>
</tr>
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<tr>
<th>Ex 3. When a polynomial ( P(x) = 3x^3 + cx^2 + dx - 7 ) is divided by ( x - 2 ), the remainder is (-3). When ( P(x) ) is divided by ( x + 1 ), the remainder is (-18). What are the values of ( c ) and ( d )?</th>
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#### B The Remainder Theorem (II)

If a polynomial \( P(x) \) is divided by \( ax - b \) then the remainder is \( r = P(b/a) \).

**Proof:**

<table>
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<tr>
<th>Ex 4. Determine the remainder when ( P(x) = 2x^3 + 3x^2 - 7x - 3 ) is divided by ( 2x + 5 ).</th>
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</table>
### C The Factor Theorem

A polynomial \( P(x) \) has \( x - b \) as a **factor** if and only if \( P(b) = 0 \).

Note. In this case \( b \) is a zero of the polynomial function \( P(x) \).

**Ex 5.** Determine whether

- a) \( x + 2 \) is a factor of \( P(x) = x^3 + 5x^2 + 2x - 8 \)
- b) \( x^2 - 1 \) is a factor of \( P(x) = 2x^4 - 3x^3 - x^2 + 3x - 1 \)

### D Integral Zero Theorem

If \( x = b \) is an **integral zero** of the polynomial \( P(x) \) with **integral coefficients**, then \( b \) is a factor (divisor) of the constant term \( a_0 \) of the polynomial.

- **c)** \( P(x) = x^4 - 2x^3 - x^2 + 4x - 2 \)
- **b)** \( P(x) = 2x^3 + 3x^2 - 3x - 2 \)

### E Rational Zero Theorem

If \( x = b / a \) is an **rational zero** of the polynomial \( P(x) \) with **integral coefficients**, then \( b \) is a factor (divisor) of the constant term \( a_0 \) and \( a \) is a factor (divisor) of the leading term \( a_n \).

**Ex 7.** Factor completely.

\[ P(x) = 12x^4 - 4x^3 - 11x^2 + x + 2 \]

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**Reading:** Nelson Textbook, Pages 171-176

**Homework:** Nelson Textbook, Page 176: #1, 2, 5, 6ab, 7af, 9, 10, 12, 13, 16