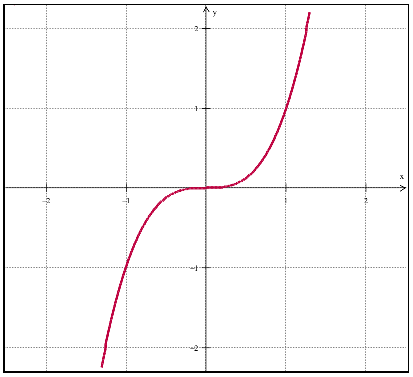
3.4 Transformations of Power Functions (Cubic, Quartic, and other)

A Cubic Function

The *cubic* function has the parent function $f(x)=x^3$ and after transformations may be written as:

$$f(x)=a[b(x-c)^3]+d$$



Ex 1. Use transformations to graph each function.

a) $f(x)=-2x^3$

b) $f(x)=(x-1)^3-2$

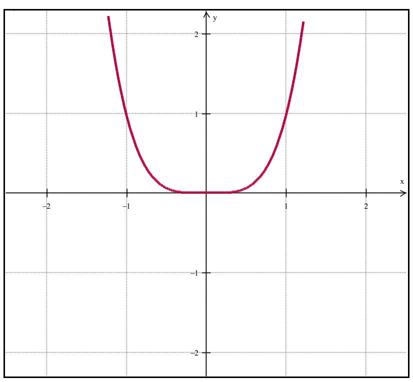
c) $f(x)=-(x+2)^3+3$

d) $f(x)=-(3-x)^3-2$

B Quartic Function

The *quartic* function has the parent function $f(x)=x^4$ and after transformations may be written as:

$$f(x)=a[b(x-c)^4]+d$$



Ex 2. Use transformations to graph each function.

a) $f(x)=-(x-2)^4$

b) $f(x)=(x+1)^4-3$

c) $f(x)=2(x-1)^4-1$

d) $f(x)=2-(3-x)^4$

Ex 3. Find the real zeros (x-intercepts) and the y-intercept.

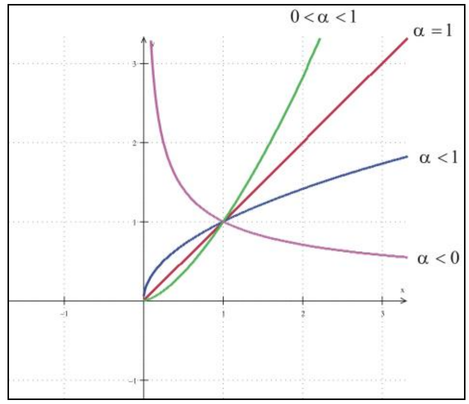
a) $f(x)=8+(x+2)^3$

b) $f(x)=16-(2x-1)^4$

C Power Function (Real Exponent)

The power function with a real exponent is defined by:

$$f(x) = x^{\alpha} \quad ; \quad \alpha \in R$$



D Power Function (Rational Exponent)

The power function with a rational exponent is defined by:

$$f(x) = x^{m/n} \quad ; \quad n \neq 0$$

where m and n are integers.

Ex 4. Use symmetry and exponent rules to sketch the graph of the following functions.

a) $y = x^{1/3}$

b) $y = x^{2/3}$

c) $y = x^{3/2}$

d) $y = x^{4/3}$

e) $y = x^{-1/3}$

f) $y = x^{-2/3}$

Ex 5. Use transformations to sketch the graph of the following functions.

a) $y = -(x+2)^{1/3}$

b) $y = 2-(x-1)^{3/4}$

c) $y = (x+4)^{-3/2}$

Ex 6. Sketch the graph of the following functions.

a) $f(x) = x^2\sqrt[3]{x-1}$

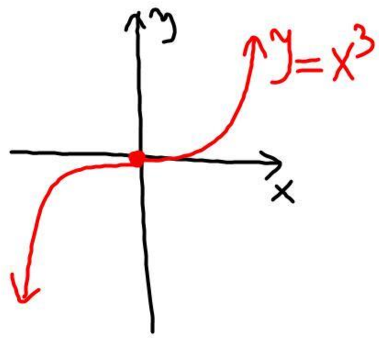
b) $f(x) = x^{1/3}(x-8)^{2/3}$

Reading: Nelson Textbook, Pages 149-155
Homework: Nelson Textbook, Page 155: #1, 3ab, 6ab, 9, 10, 14

$f(x) = a\left[b(x-c)\right]^3 + d$

$\uparrow$

$f(x) = x^3$



$\underline{\underline{a}}$ If $a<0$ (Vertical Reflection in the x-axis)

If $|a|>1$ (Vertical Expansion (stretch) by a factor of $|a|$)

If $|a|<1$ (Vertical Compression by a factor of $|a|$)

$\underline{\underline{b}}$ If $b<0$ (Reflection in the y-axis)

If $|b|>1$ (Horizontal Compression by a factor of $\frac{1}{|b|}$)

If $|b|<1$ (Horizontal Expansion (Stretch) by a factor of $\frac{1}{|b|}$)

$$\text{Scale Factor} = \frac{\text{image}}{\text{original}}$$

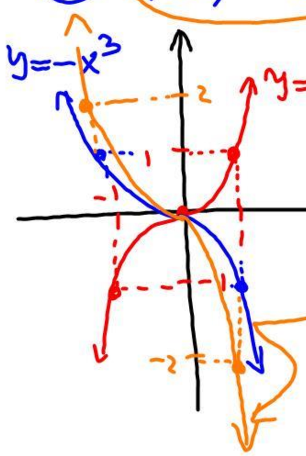
$\underline{\underline{c}}$ If $c > 0$ (Horizontal Translation (Shift)
by $|c|$ units to the Right)

If $c < 0$ (Horizontal Translation (Shift)
by $|c|$ units to the Left

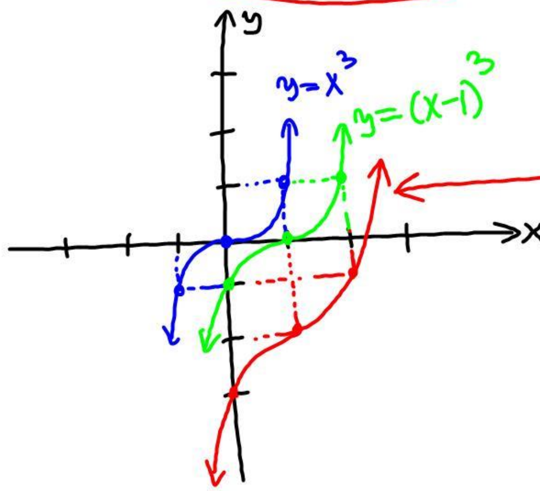
$\underline{\underline{d}}$ If $d > 0$ (Vertical Translation (Shift)
by $|d|$ units up (upward)

If $d < 0$ (Vertical Translation (Shift)
by $|d|$ units down(ward)

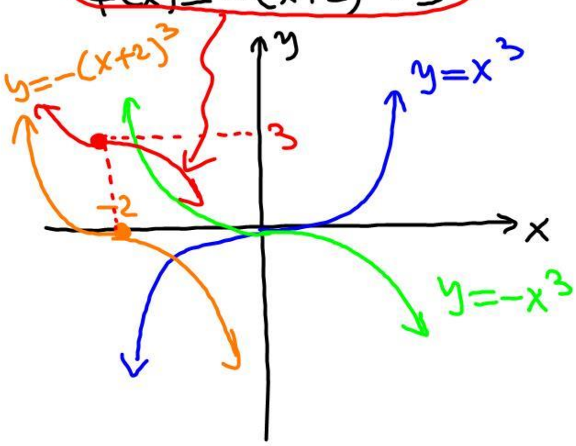
1a $f(x) = -2x^3$



1b $f(x) = (x-1)^3 - 2$



1c $f(x) = -(x+2)^3 + 3$



1d $f(x) = -(3-x)^3 - 2$

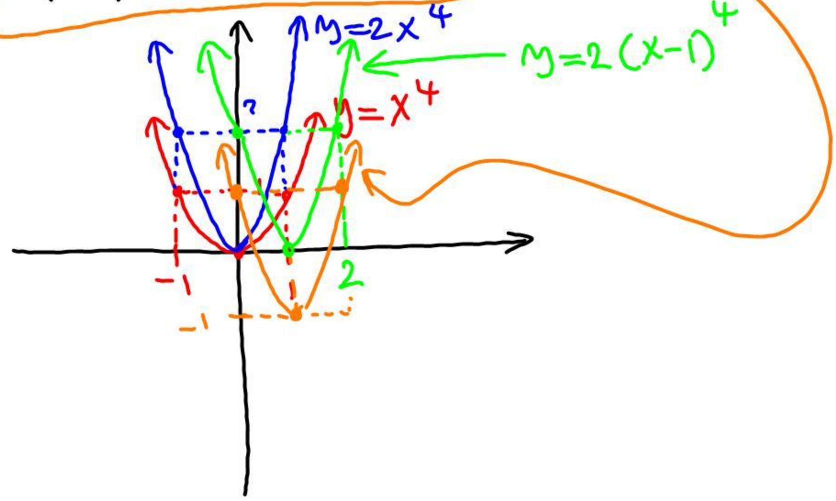
$= (x-3)^3 - 2$

TR3

VT↓2

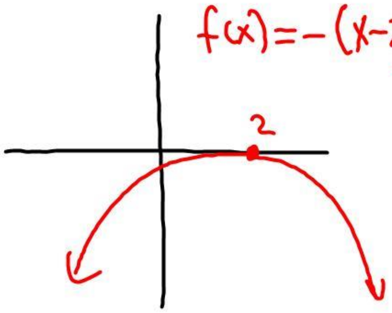
2c

$f(x) = 2(x-1)^4 - 1$



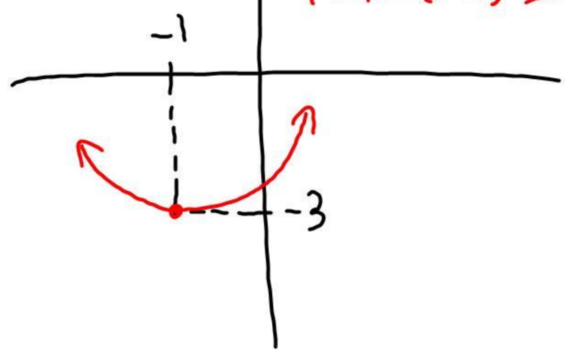
2a

$f(x) = -(x-2)^4$



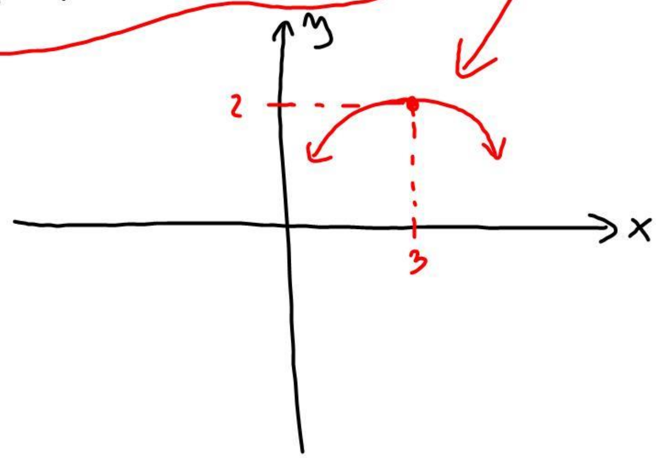
2b

$f(x) = (x+1)^4 - 3$



2d

$$f(x) = -(x-3)^4 + 2$$



3a

$$f(x) = 8 + (x+2)^3$$

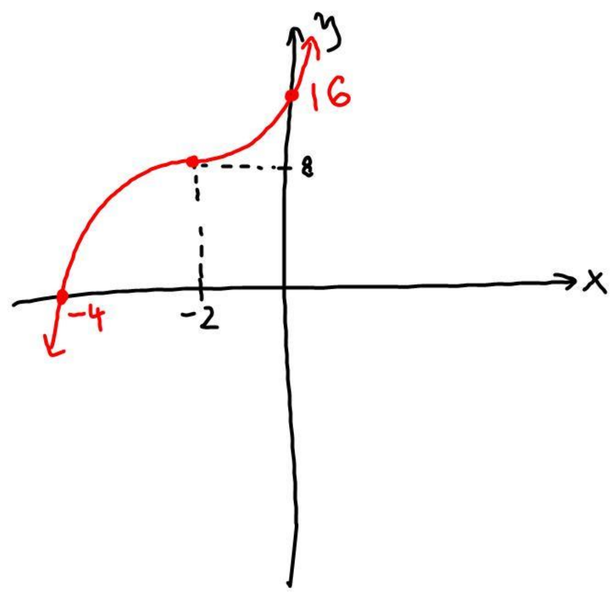
$$f(0) = 16 = y_{int}$$

$$0 = 8 + (x+2)^3$$

$$-8 = (x+2)^3$$

$$-2 = x+2$$

$$x = -4 = x_{int}$$



3b

$$f(x) = 16 - (2x-1)^4$$

$$y_{int} = f(0) = 15$$

<u>x_{int}</u>

$$0 = 16 - (2x-1)^4$$

$$(2x-1)^4 = 16$$

$$2x - 1 = \pm 2$$

$$2x = 1 \pm 2 \rightarrow 3,\ -1$$

$$x_{int} = -\tfrac{1}{2}, \tfrac{3}{2}$$

$$f(x) = -16\left(x - \tfrac{1}{2}\right)^4 + 16$$