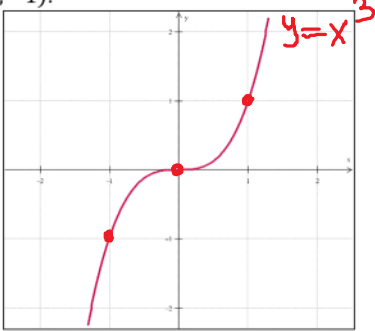
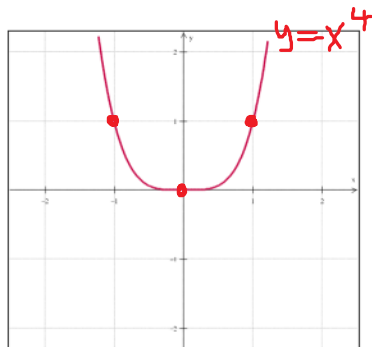


3.4 Transformations of Power Functions (Cubic, Quartic, and other)

<p>A Transformations</p> <p>Two functions may be related by transformations. Let $g(x) = a[b(x - c)] + d$. Then, the graph of the function $y = g(x)$ may be obtained from the graph of the function $y = f(x)$ by performing the following transformations:</p> <ul style="list-style-type: none"> The graph is scaled vertically by a factor of a. If $a < 0$ then the graph is also <u>reflected</u> in the <u>x-axis</u>. If $a > 1$ then the graph is <u>expanded vertically</u> and if $a < 1$ then the graph <u>compressed vertically</u>. The graph is <u>translated vertically upward</u> if $d > 0$ or <u>downward</u> if $d < 0$ by d units. 	<ul style="list-style-type: none"> The graph of is scaled horizontally by a factor of $1/ b$. If $b < 0$ then the graph is also <u>reflected</u> in the <u>y-axis</u>. The graph is <u>translated horizontally to the right</u> if $c > 0$ or to the <u>left</u> if $c < 0$ by c units. The graph is <u>horizontally compressed</u> if $b > 1$ and is <u>horizontally expanded</u> if $b < 1$.
<p>B Cubic Function</p> <p>A cubic function has the parent function $f(x) = x^3$ and after transformations may be written as:</p> $f(x) = a[b(x - c)]^3 + d$ <p>Here is the graph of the original (parent) function $f(x) = x^3$ and the key points are $(0,0)$, $(1,1)$, and $(-1,-1)$.</p> 	<p>Ex 1. Use transformations to graph each function.</p> <p>a) $f(x) = -2x^3$</p> <p>b) $f(x) = (x - 1)^3 - 2$</p> <p>c) $f(x) = -(x + 2)^3 + 3$</p> <p>d) $f(x) = -(3 - x)^3 - 2$</p>
<p>C Quartic Function</p> <p>A quartic function has the parent function $f(x) = x^4$ and after transformations may be written as:</p> $f(x) = a[b(x - c)]^4 + d$	<p>Ex 2. Use transformations to graph each function.</p> <p>a) $f(x) = -(x - 2)^4$</p> <p>b) $f(x) = (x + 1)^4 - 3$</p>

Here is the graph of the original (parent) function $f(x) = x^4$ and the key points are $(0,0)$, $(1,1)$, and $(-1,1)$.

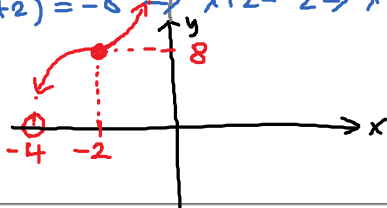


c) $f(x) = 2(x-1)^4 - 1$

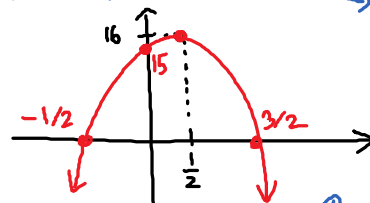
d) $f(x) = 2 - (3-x)^4$

Ex 3. Find the real zeros (x-intercepts) and the y-intercept. Graph the function.

a) $f(x) = 8 + (x+2)^3 = y$
 $y\text{-int} = f(0) = 8 + 8 = 16$
 $x\text{-int} \Rightarrow y = 0 \Rightarrow 8 + (x+2)^3 = 0$
 $(x+2)^3 = -8 \Rightarrow x+2 = -2 \Rightarrow x\text{-int} = -4$



b) $f(x) = 16 - (2x-1)^4 = y$
 $y\text{-int} = f(0) = 16 - 1 = 15$
 $x\text{-int} \Rightarrow y = 0 \Rightarrow 16 - (2x-1)^4 = 0 \Rightarrow (2x-1)^4 = 16 \Rightarrow 2x-1 = \pm 2 \Rightarrow x = 3/2, x = -1/2$



Ex 4. Sketch the graph of the following polynomial functions.

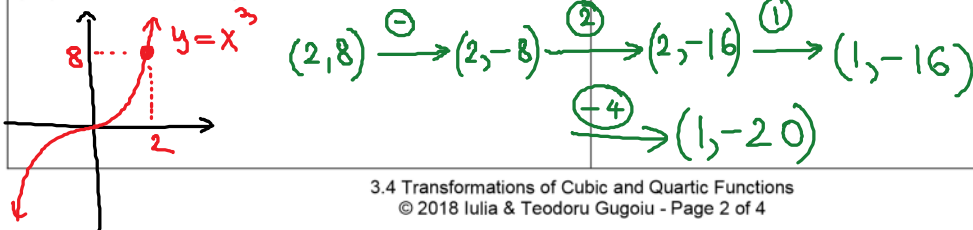
a) $f(x) = x^3 + x^2 + 2$
 $y = x^3 + x^2 + 2$
 $y = x^2(x+1) + 2$
 $y = x^2 + x^2 + 2$

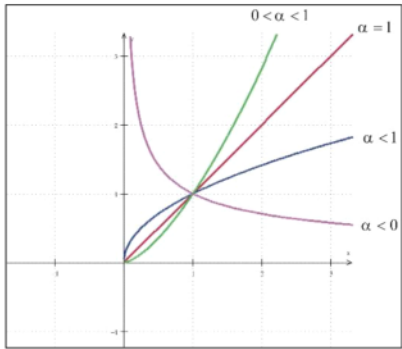
b) $f(x) = x^4 - 4x^2 - 3$
 $x^2(x^2 - 4) = x^2(x-2)(x+2)$
 $y = x^4 - 4x^2 - 3$

c) $f(x) = x^3 - 9x + 4$
 $y = x^3 - 9x + 4$
 $y = x^2 - 9x$

d) $f(x) = x^4 + x^3 - 5$
 $y = x^4 + x^3 - 5$
 $y = x^4 + x^3 - 5$

Ex 5. Find the corresponding point on the graph of the function $y = -2(x+1)^3 - 4$ to the point $(2,8)$ on the graph of the function $y = x^3$.

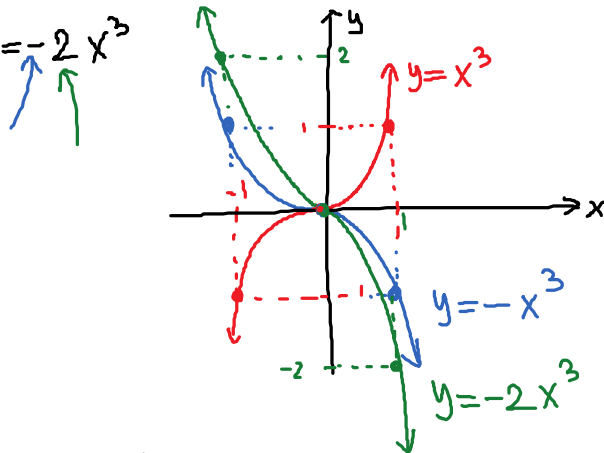


<p>D Power Function (Real Exponent)</p> <p>The power function with a real exponent is defined by:</p> $f(x) = x^\alpha \quad ; \quad \alpha \in R$	
<p>E Power Function (Rational Exponent)</p> <p>The power function with a rational exponent is defined by:</p> $f(x) = x^{m/n} \quad ; \quad n \neq 0$ <p>where m and n are integers.</p>	<p>Ex 6. Use symmetry and exponent rules to sketch the graph of the following functions.</p> <p>a) $y = x^{1/3}$</p> <p>b) $y = x^{2/3}$</p> <p>c) $y = x^{3/2}$</p> <p>d) $y = x^{4/3}$</p> <p>e) $y = x^{-1/3}$</p> <p>f) $y = x^{-2/3}$</p>
<p>Ex 7. Use transformations to sketch the graph of the following functions.</p> <p>a) $y = -(x+2)^{1/3}$</p> <p>b) $y = 2 - (x-1)^{3/4}$</p> <p>c) $y = (x+4)^{-3/2}$</p>	<p>Ex 8. Sketch the graph of the following functions.</p> <p>a) $f(x) = x^2 \sqrt[3]{x-1}$</p> <p>b) $f(x) = x^{1/3} (x-8)^{2/3}$</p>

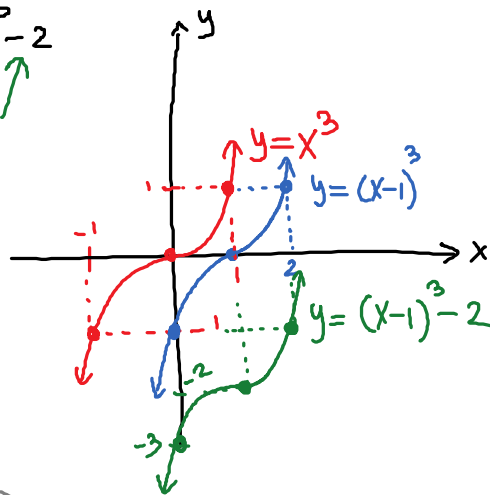
Reading: Nelson Textbook, Pages 149-155

Homework: Nelson Textbook, Page 155: #1, 3ab, 6ab, 9, 10, 14

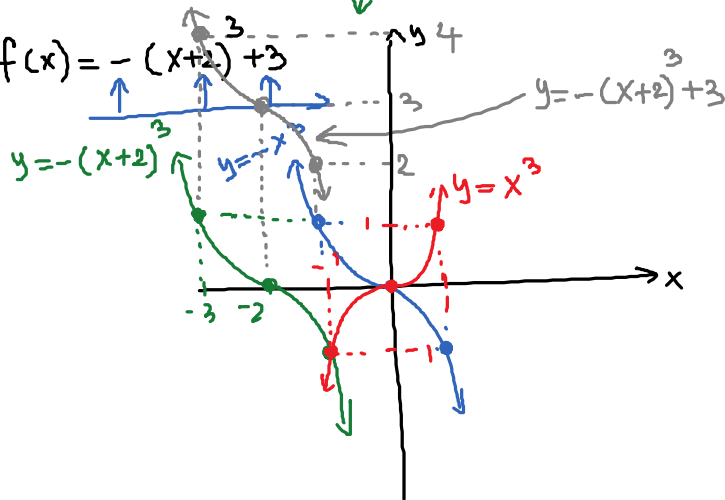
1a) $f(x) = -2x^3$



1b) $f(x) = (x-1)^3 - 2$

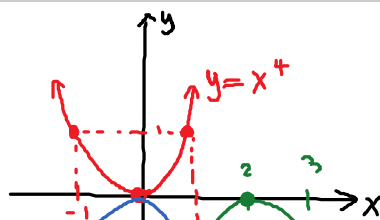


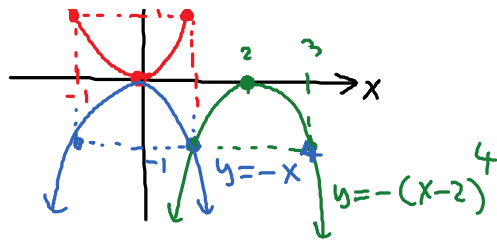
1c) $f(x) = -(x+2)^3 + 3$



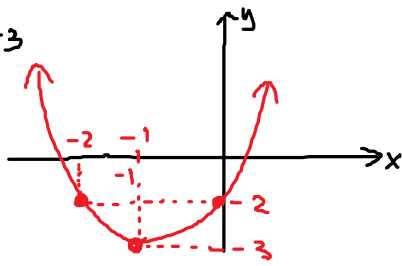
3.4 Transformations of Cubic and Quartic Functions
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2a) $f(x) = -(x-2)^4$





2b) $f(x) = (x+1)^4 - 3$



2c) $f(x) = 2(x-1)^4 - 1$

