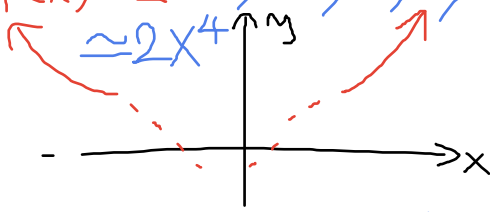



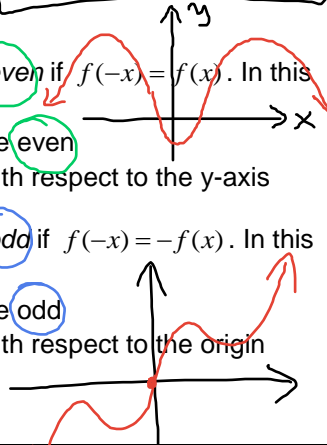
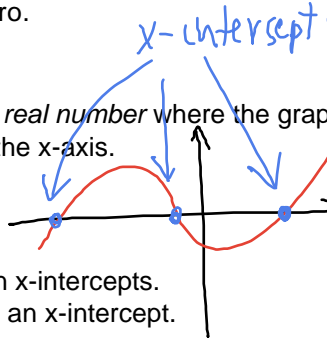




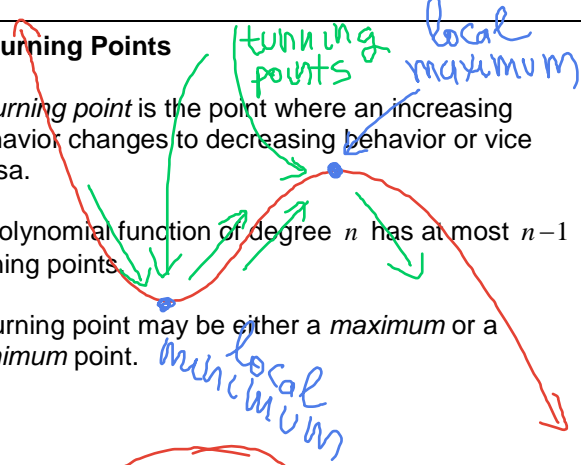
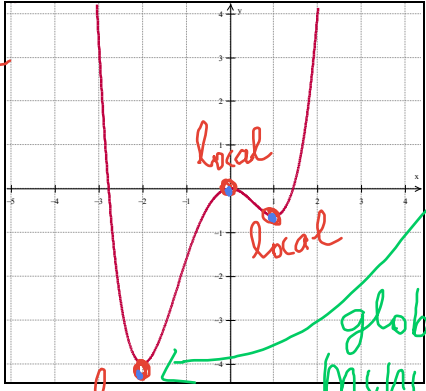
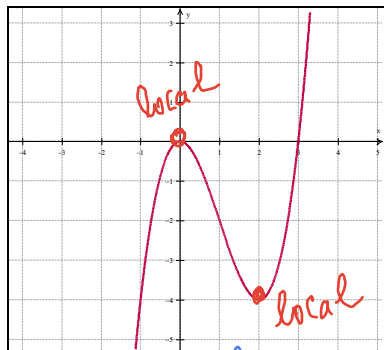


3.2 Characteristics of Polynomial Functions

<p>A End Behaviour</p> <p>For x very large ($x \rightarrow \infty$ or $x \rightarrow -\infty$) the graph of the polynomial function resemble the graph of the leading term $y = a_n x^n$.</p>	<p>Ex 1. For each case, find an example and sketch the end behaviour of the polynomial function.</p>
<p>1a) $a_n > 0$, even n</p> <p>Ex $f(x) = 2x^4 - x^3 + x^2 - x + 1$ $\approx 2x^4$</p> 	<p>1b) $a_n > 0$, odd n</p> <p>Ex $f(x) = x^5 + 3x^4 - \dots$ $\approx x^5$</p> 
<p>1c) $a_n < 0$, even n</p> <p>$f(x) = -x^2 + 2x - 7$ $\approx -x^2$</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>As $x \rightarrow \pm \infty$ $f(x) \rightarrow -\infty$</p> </div>	<p>1d) $a_n < 0$, odd n</p> <p>$f(x) = -2x^3 + x^2 - 2x + 1$ $\approx -2x^3$</p>  <div style="border: 1px solid black; padding: 5px; width: fit-content; margin: 10px auto;"> <p>As $x \rightarrow \infty$, $f(x) \rightarrow -\infty$ As $x \rightarrow -\infty$, $f(x) \rightarrow +\infty$</p> </div>
<p>B Symmetry</p> <p>A polynomial function is even if $f(-x) = f(x)$. In this case:</p> <ul style="list-style-type: none"> all exponents of x are even graph is symmetric with respect to the y-axis <p>A polynomial function is odd if $f(-x) = -f(x)$. In this case:</p> <ul style="list-style-type: none"> all exponents of x are odd graph is symmetric with respect to the origin 	<p>Ex 2. Classify as even, odd, or neither.</p> <p>a) $f(x) = x - x^3 + 2x^5$ odd</p> <p>b) $f(x) = 2 - x^2 + 3x^6$ even $2 = 2x^0$</p> <p>c) $f(x) = 1 + x - x^3 + 3x^4$ neither</p> <p>d) $f(x) = (x^2 - 1)^3$ even $f(-x) = [(-x)^2 - 1]^3 = (x^2 - 1)^3 = f(x)$</p> <p>e) $f(x) = -x^3(x^2 + 1)^2$ odd</p>
<p>C Zeros versus x-intercepts</p> <p>A zero z is a number (real or complex) where the value of the function is zero. So $y = f(z) = 0$.</p> <p>An x-intercept x-int is a real number where the graph of the function intersects the x-axis. So $y = f(x - \text{int}) = 0$.</p>  <p>Notes.</p> <ul style="list-style-type: none"> A real zero is also an x-intercept. A complex zero is not an x-intercept. 	<p>Ex 3. (Optional) Find the zeros and the x-intercepts for $f(x) = (x-1)(x^2+1)$</p> <p>Zeros: $1, +i, -i$</p> <p>$i^2 = -1$ $i^2 + 1 = 0$</p> <p>x-intercepts: 1</p>

<p>D Fundamental Theorem of Algebra</p> <p>A polynomial function $y = P(x)$ of degree n has n zeros (real or complex).</p> <p>Notes:</p> <ul style="list-style-type: none"> If the <i>coefficients</i> of the polynomial function are <i>real numbers</i> then complex zeros come in conjugate pairs (the number of complex zeros may be 0 (none), 2, 4, 6, ...) The number of <i>real zeros</i> is <i>at most</i> n. These zeros may be distinct (different) or coincident (same) A polynomial function of <i>even degree</i> <i>may have no real zero</i> (all may be complex). A polynomial function of <i>odd degree</i> must have <i>at least one real zero</i>. 	<p>Ex 4. Analyze the general shape for the following polynomial functions.</p> <p>a) linear </p> <p>b) quadratic </p> <p>c) cubic </p> <p>d) quartic </p>
<p>E Turning Points</p> <p>A <i>turning point</i> is the point where an increasing behavior changes to decreasing behavior or vice versa.</p> <p>A polynomial function of degree n has at most $n-1$ turning points</p> <p>A turning point may be either a <i>maximum</i> or a <i>minimum</i> point.</p> 	<p>F Extrema Points</p> <p>Extrema is the plural of extremum. Extremum is either a minimum or a maximum. An extremum may be local (relative) or global (absolute). Each turning point is a local extremum point. A polynomial function of even degree has either a global (absolute) minimum or a global (absolute) maximum point. A polynomial function of odd degree has neither a global (absolute) minimum nor a global (absolute) maximum point.</p>
<p>Ex 5. Find the <u>turning points</u> for the polynomial function represented below. What degree may have this polynomial function? Is this degree odd or even?</p> <p>n may be 4, 6, 8,</p> 	<p>Ex 6. Find the <u>local (relative)</u> and <u>global (absolute)</u> extremum points for the polynomial function at example 5.</p> <p>Ex 7. Find the local (relative) and global (absolute) extremum points for the polynomial function represented below.</p>  <p>No global extrema</p>

Reading: Nelson Textbook, Pages 129-135

Homework: Nelson Textbook, Page 136: #1, 3, 5, 7, 10, 11, 14, 16