

MHF4U

9.2 Characteristics of Polynomial Functions

1. Find if the following polynomial functions are odd or even.

a)  $f(x) = 1 - x^2$  even

b)  $f(x) = x - 2x^3$  odd

c)  $f(x) = x + x^4 - x^3 - 2$  neither

d)  $f(x) = (x^3 - x)^2$

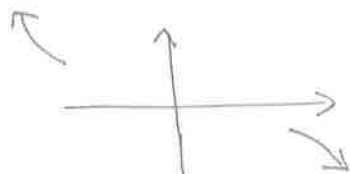
$f(-x) = [(-x)^3 - (-x)]^2 = [-x^3 + x]^2 = [-(x^3 - x)]^2 = (x^3 - x)^2 = f(x)$   
 $\therefore$  even

e)  $f(x) = (x^2 + 1)^2(x^3 + x)$

$f(-x) = [(-x)^2 + 1]^2 [(-x)^3 + (-x)] = (x^2 + 1)^2 [-x(x^2 + 1)] = -f(x) \Rightarrow \therefore$  odd

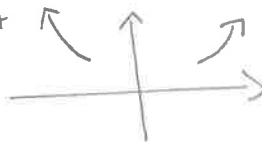
2. Analyse the end behaviour of each polynomial function.

a)  $f(x) = -2x^3 + 3x - 1$  behaves as  $-2x^3$



As  $x \rightarrow \infty, y \rightarrow -\infty$   
 As  $x \rightarrow -\infty, y \rightarrow \infty$

b)  $f(x) = 3x^4 - 3x^3 + x - 4$  behaves as  $3x^4$



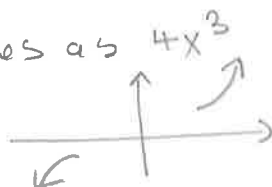
As  $x \rightarrow \pm \infty$   
 $y \rightarrow \infty$

c)  $f(x) = (1 - x^2)(x^3 + 1)$  behaves as  $-x^5$



As  $x \rightarrow \infty, y \rightarrow -\infty$   
 As  $x \rightarrow -\infty, y \rightarrow \infty$

d)  $f(x) = x(1 - 2x)^2$  behaves as  $4x^3$



As  $x \rightarrow \pm \infty, y \rightarrow \pm \infty$

3. If  $P(x)$  is even and  $Q(x)$  is odd, what can you say about the following compositions? Are they odd or even? Explain. Find examples to prove you are right.

a)  $f(x) = P(x) + Q(x)$

$f(-x) = P(-x) + Q(-x)$   
 $= P(x) - Q(x)$   
 $\neq f(x)$   
 $\neq -f(x)$

$\therefore$  neither

Ex:  $f(x) = x^2 + x$

a)  $g(x) = P(x)Q(x)$

$g(-x) = P(-x)Q(-x)$   
 $= P(x)[-Q(x)]$   
 $= -P(x)Q(x)$   
 $= -g(x)$

$\therefore$  odd

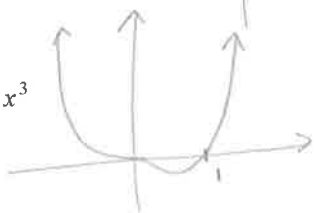
Ex:  $g(x) = x^2(x) = x^3$

4. Use Desmos to compare the graph of the following functions. How many turning points are there?

a)  $f(x) = x^4$



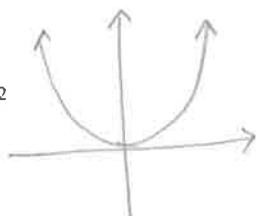
b)  $f(x) = x^4 - x^3$



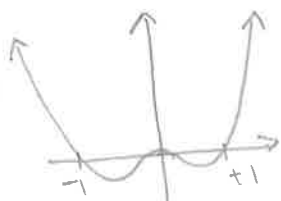
c)  $f(x) = x^4 + x^3$



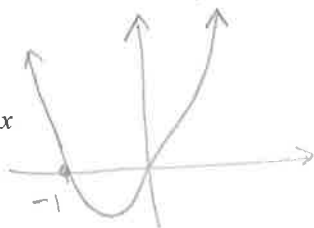
d)  $f(x) = x^4 + x^2$



e)  $f(x) = x^4 - x^2$



f)  $f(x) = x^4 + x$



g)  $f(x) = x^4 - x$

