3.1 Exploring Polynomial Functions

A Polynomial Functions

x $x^0=1$

A *polynomial function* $y = f(x)$ is defined by:

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0 x^0$$

where:

- $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are *real numbers* called the *coefficients* of the polynomial function
- a_n is called *leading coefficient*
- $a_n x^n$ is called *leading term*
- a_0 is called the *constant term*
- n is a *non-negative integer* that gives the *degree* of the polynomial function

Note. The degree of the polynomial function n is the largest exponent of x

Ex 1. Verify if the following expressions are or not polynomial functions.

a) $f(x) = \sqrt{2}x^3 - 2x^2$ yes $n=3$

b) $f(x) = 2\sqrt{x} - x^2$ $\sqrt{x} = x^{\frac{1}{2}}$ no

c) $f(x) = x^2 + \frac{1}{x} = x^2 + x^{-1}$ no

d) $f(x) = (x-1)(x+2)^2$ yes

B Order

The terms of a polynomial function can be written in any order because the addition operation is a commutative operation.

$CT = 3$

Ex 2. Consider $f(x) = x - 2x^3 - 4x^2 + 3 - x^4$ $n=4$

a) Is this function polynomial? yes If yes, find the degree, the leading term, the leading coefficient, and the constant term

$LT = -x^4$ $LC = -1$

b) write the polynomial function in order of increasing powers of the variable x

$f(x) = 3 + x - 4x^2 - 2x^3 - x^4$

c) write the polynomial function in order of decreasing powers of the variable x

$f(x) = -x^4 - 2x^3 - 4x^2 + x + 3$

C Specific Polynomials

If $n = 0$, $f(x) = a_0$ is called *constant* function.

If $n = 1$, $f(x) = a_1 x + a_0$ is called *linear* function.

If $n = 2$, $f(x) = a_2 x^2 + a_1 x + a_0$ is called *quadratic* function.

If $n = 3$, $f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$ is called *cubic* function.

Note. For $n = 4$ we have the *quartic* function and for $n = 5$ we have the *quintic* function.

Ex 3. Identify each polynomial function as constant, linear, quadratic, cubic, quartic, or quintic.

a) $f(x) = -2$ constant

b) $f(x) = -x^2 + 3$ quadratic

c) $f(x) = 2x^3 - 3x^2 + x$ cubic

d) $f(x) = 2 - 3x$ linear

e) $f(x) = x^5 + x^3$ quintic

f) $f(x) = 1 - x^2 - x^4 + x$

D Operations with polynomial functions

All the four operations (addition, subtraction, multiplication, and division) are defined for polynomial functions.

$= (6x-3x^2)(x-2)$

$= 6x^2 - 12x - 3x^3 + 6x^2$

$= -3x^3 + 12x^2 - 12x$

Ex 4. Consider two polynomial functions $f(x) = 6x - 3x^2$ and $g(x) = x - 2$. Do the required operations:

a) $f(x) + g(x) = 6x - 3x^2 + x - 2$
$= -3x^2 + 7x - 2$

b) $f(x) - g(x)$
$= (6x - 3x^2) - (x - 2) = -3x^2 + 5x + 2$

c) $f(x)g(x)$

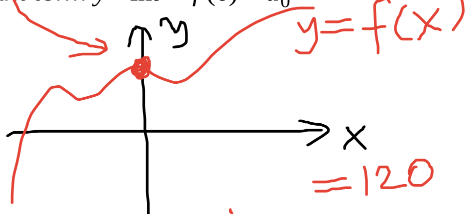
d) $f(x)/g(x) = \dfrac{6x-3x^2}{x-2} = \dfrac{3x\cancel{(2-x)}^{-1}}{\cancel{x-2}} = -3x$

$x \neq 2$

E y-intercept

y-int = f(0)

The y-intercept of a polynomial function is equal with the constant term $y - \text{int} = f(0) = a_0$



Ex 5. Find the y-intercept for each polynomial function.

a) $f(x) = -2$ → y-int = −2

b) $f(x) = -x^2 + 3$ → 3

c) $f(x) = 2x^3 - 3x^2 + x$ → 0

d) $f(x) = (x^2 + 1)(x - 2)$ → −2

e) $f(x) = (x^3 - 2)^3$ → −8

f) $f(x) = -2(x+3)^2(x-1)^5$ → $-2(3)^2(-1)^5 = -2(9)(-1) = 18$

F Finite Differences

$5! = 5(4)(3)(2)(1)$

$= 120$

The *nth* finite differences of a polynomial function of degree n are constant.
This constant c is related to a_n and n by:

$$\boxed{c = n!\,a_n}$$

$a_n = \dfrac{-48}{n!} = \dfrac{-48}{4(3)(2)} = -2$

where $n!$ (n factorial) is defined by

$$n! = 1 \times 2 \times 3 \times \ldots \times (n-1) \times n$$

Note: Use "following # minus preceding #" rule to find the differences:

a		
	$b-a$	
b		$(c-b)-(b-a)$
	$c-b$	
c		

Ex 6. Use the information provided bellow and the finite differences method to find the *degree* of the polynomial function and the *leading coefficient*.

→ n = 4

→ $a_n = -2$

x	y	$\Delta^1 y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-4	-476	342	-218	120	-48	
-3	-134	124	-98	72	-48	
-2	-10	26	-26	24	-48	
-1	16	0	-2	-24	-48	
0	16	-2	-26	-72	-48	
1	14	-28	-98	-120		
2	-14	-126	-218			
3	-140	-344				
4	-484					

n = 4 (quartic)

Reading: Nelson Textbook, Pages 124-126
Homework: Nelson Textbook, Page 127: #1, 2, 5