3.1 Exploring Polynomial Functions

A Polynomial Functions

A *polynomial function* $y = f(x)$ is defined by:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

where:

- $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are *real numbers* called the *coefficients* of the polynomial function
- a_n is called *leading coefficient*
- $a_n x^n$ is called *leading term*
- a_0 is called the *constant term*
- n is a *non-negative integer* that gives the *degree* of the polynomial function

Note. The degree of the polynomial function n is the largest exponent of x

Ex 1. Verify if the following expressions are or not polynomial functions.

a) $f(x) = \sqrt{2}x^3 - 2x^2$

b) $f(x) = 2\sqrt{x} - x^2$

c) $f(x) = x^2 + \frac{1}{x}$

d) $f(x) = (x-1)(x+2)^2$

B Order

The terms of a polynomial function can be written in any order because the addition operation is a commutative operation.

Ex 2. Consider $f(x) = x - 2x^3 - 4x^2 + 3 - x^4$

a) Is this function polynomial? If yes, find the degree, the leading term, the leading coefficient, and the constant term

b) write the polynomial function in order of increasing powers of the variable x

c) write the polynomial function in order of decreasing powers of the variable x

C Specific Polynomials

If $n = 0$, $f(x) = a_0$ is called *constant* function.
If $n = 1$, $f(x) = a_1 x + a_0$ is called *linear* function.
If $n = 2$, $f(x) = a_2 x^2 + a_1 x + a_0$ is called *quadratic* function.
If $n = 3$, $f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$ is called *cubic* function.

Note. For $n = 4$ we have the *quartic* function and for $n = 5$ we have the *quintic* function.

Ex 3. Identify each polynomial function as constant, linear, quadratic, cubic, quartic, or quintic.

a) $f(x) = -2$

b) $f(x) = -x^2 + 3$

c) $f(x) = 2x^3 - 3x^2 + x$

d) $f(x) = 2 - 3x$

e) $f(x) = x^5 + x^3$

f) $f(x) = 1 - x^2 - x^4 + x$

D Operations with polynomial functions

All the four operations (addition, subtraction, multiplication, and division) are defined for polynomial functions.

Ex 4. Consider two polynomial functions $f(x)=6x-3x^2$ and $g(x)=x-2$. Do the required operations:

a) $f(x)+g(x)$

b) $f(x)-g(x)$

c) $f(x)g(x)$

d) $f(x)/g(x)$

E y-intercept

The y-intercept of a polynomial function is equal with the constant term $y-\text{int}=f(0)=a_0$

Ex 5. Find the y-intercept for each polynomial function.

a) $f(x)=-2$

b) $f(x)=-x^2+3$

c) $f(x)=2x^3-3x^2+x$

d) $f(x)=(x^2+1)(x-2)$

e) $f(x)=(x^3-2)^3$

f) $f(x)=-2(x+3)^2(x-1)^5$

F Finite Differences

The *nth* finite differences of a polynomial function of degree n are constant.
This constant c is related to a_n and n by:

$$c=n!a_n$$

where $n!$ (n factorial) is defined by

$$n!=1\times 2\times 3\times\ldots\times(n-1)\times n$$

Note: Use "following # minus preceding #" rule to find the differences:

a
$\quad b-a$
b
$\quad c-b$
c

Ex 6. Use the information provided bellow and the finite differences method to find the *degree* of the polynomial function and the *leading coefficient*.

x	y	$\Delta^1 y$	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
-4	-476					
-3	-134					
-2	-10					
-1	16					
0	16					
1	14					
2	-14					
3	-140					
4	-484					

Reading: Nelson Textbook, Pages 124-126
Homework: Nelson Textbook, Page 127: #1, 2, 5

