

MHF4U 3.1 Exploring Polynomial Functions
Practice Questions

1. Are the following functions polynomial? If yes, find the degree and the name of the polynomial function.

a) $f(x) = x + x^{-1} + x^{1/2}$ No ; -1 and $1/2$ are not positive integers

b) $f(x) = \frac{x^2 + x}{x}$ No ; f is not defined at $x=0$

c) $f(x) = (x^2 + 1)(x - 1)$ Yes ; $n=3$; cubic function

d) $f(x) = (x^3 - x)^2$ Yes ; $n=6$

f) $f(x) = 1 - \sqrt{x^2}$ No ; $\sqrt{x^2} = |x| \neq x$

2. Are the following functions polynomial? If yes, find the leading coefficient and the constant term.

a) $f(x) = (x^2 - 1)^3$ Yes ; $n=6$; $a_6 = 1$; $a_0 = -1$

b) $f(x) = (x^2 - 1)^2(x + 2)^3$ Yes ; $n=7$; $a_7 = 1$; $a_0 = 8$

c) $f(x) = (x + 1)^4 - (x - 1)^4$ Yes ; $n=3$; $a_3 = 8$; $a_0 = 0$

d) $f(x) = x(x - 1)(x + 2)(x - 3)$ Yes ; $n=4$; $a_4 = 1$; $a_0 = 0$

3. If the y-intercept of the polynomial function $P(x)$ is 3 and the y-intercept of the polynomial function $Q(x)$ is -2 , what is the y-intercept of the polynomial function $f(x) = [P(x) - Q^2(x)]^3$?

$$\begin{aligned} \text{y-int} = f(0) &= [P(0) - Q^2(0)]^3 \\ &= [3 - (-2)^2]^3 \\ &= (-1)^3 \\ &= -1 \end{aligned}$$

4. Are the following relations polynomial? If yes, find the degree and the leading coefficient.

a)

x	y	Δ^1	Δ^2	Δ^3
-5	325	114	54	-12
-4	176	110	42	-12
-3	81	105	30	-12
-2	28	90	18	-12
-1	5	75	6	-12
0	0	60	-6	-12
1	1	45	-18	-12
2	-4	30	-30	-12
3	-27	15	-42	-12
4	-80	0	-54	-12
5	-175			

$n=3$
 $a_3 = \frac{-12}{3!} = -2$

b)

x	y	Δ^1	Δ^2
-5	24	11	2
-4	15	10	2
-3	8	9	2
-2	3	8	2
-1	0	7	2
0	-1	6	2
1	0	5	2
2	3	4	2
3	8	3	2
4	15	2	2
5	24	1	2

$n=2$
 $a_2 = \frac{2}{2!} = 1$

c)

x	y	Δ^1	Δ^2	Δ^3	Δ^4
-5	-2055	191	612	258	-72
-4	-864	157	354	186	-72
-3	-285	123	168	114	-72
-2	-60	89	54	42	-72
-1	-3	55	-18	-30	-72
0	0	21	-54	-102	-72
1	-9	-19	-108	-174	-72
2	-60	-81	-162	-246	-72
3	-255	-163	-216	-318	-72
4	-768	-245	-270	-390	-72
5	-1845	-327	-324	-462	-72

$n=4$
 $a_4 = \frac{-72}{4!} = \frac{-72}{24} = -3$

5. Show that the functions $f(x) = x^3$ and $g(x) = x^3 - x^2$ have same third order differences. Why do you think this happens.

x	x^3	Δ^1	Δ^2	Δ^3
-3	-27	19	-12	6
-2	-8	7	6	6
-1	-1	-1	6	6
0	0	-1	6	6
1	1	7	12	6
2	8	19	18	6
3	27	31	24	6

x	$x^3 - x^2$	Δ^1	Δ^2	Δ^3
-3	-36	24	-14	6
-2	-12	10	-8	6
-1	-2	2	-2	6
0	0	0	4	6
1	0	4	10	6
2	4	14	18	6
3	18	24	24	6

The third order differences of x^2 are 0.
 The second order differences of x^2 are constant.