3.1 Exploring Polynomial Functions

b) $f(x) = \sqrt{(x^2+1)^2}$
$= x^2+1$
Yes.

Challenge Questions

1. Are the following functions polynomial? Justify your answer.

a) $f(x) = x - (x^{1/3})^6$

$f(x) = x - x^2$
Yes.

b) $f(x) = \log e^{x^2+x}$

$f(x) = (x^2+x)\log e$
Yes.

2. Find the leading term of the following polynomial function.

$f(x) = -(1-2x)^3(3x-1)^2(1-x+2x^2)$

$LT = -(-2)^3(3)^2(2) = 432$

3. If the y-intercept of the polynomial function $p(x)$ is -3 and the y-intercept of the polynomial function $q(x)$ is 2, find the y-intercept of the function $f(x) = (p^2(x) - 2q(x) + 1)^2$.

$f(0) = (p^2(0) - 2q(0) + 1)^2$
$= ((-3)^2 - 2(2) + 1)^2 = 36$

4. If the degree of the polynomial function $p(x)$ is 2 and the degree of the polynomial function $q(x)$ is 3, find the degree of the polynomial function $f(x) = (p^2(x) - q^2(x))^2$.

Degree of $p^2(x)$ is 4
Degree of $q^2(x)$ is 6
Degree of $p^2(x) - q^2(x)$ is 6
Degree of $f(x)$ is 12

5. If the leading term of the polynomial function $p(x)$ is $-2x^2$ and the leading term of the polynomial function $q(x)$ is x^3, find the leading term of the polynomial function $f(x) = (p(x) + q(x))^2 - p^3(x)$.

LT of $p(x) + q(x)$ is x^3
LT of $(p(x) + q(x))^2$ is x^6
LT of $p^3(x) = -8x^6$
$\therefore$ LT of $f(x)$ is $-7x^6$

6. Find the leading term of the following polynomial function given by a table of values.

x	y	Δ_1	Δ_2	Δ_3
-3	80			
		-53		
-2	27		30	
		-23		-12
-1	4		18	
		-5		-12
0	-1		6	
		1		-12
1	0		-6	
		-5		-12
2	-5		-18	
		-23		
3	-28			

$LT = \frac{\Delta_3}{3!} = \frac{-12}{3 \times 2 \times 1} = -2$

$\therefore LT = -2$

$x_2 - x_1 = x_3 - x_2 = x_4 - x_3 = 1$

7. Prove that the 3rd order finite differences of any cubic polynomial function in the form $f(x) = ax^3 + bx^2 + cx + d$ are constant and equal to $(3\times2\times1)a$.

x_1	$ax_1^3 + bx_1^2 + cx_1 + d$	$a(x_1^2 + x_1x_2 + x_2^2) + b(x_1 + x_2) + c$	$a(x_1 + x_2) + ax_2(2) + b(2)$	$6a$
x_2	$ax_2^3 + bx_2^2 + cx_2 + d$	$a(x_2^2 + x_2x_3 + x_3^2) + b(x_2 + x_3) + c$	$a(x_2 + x_3) + ax_3(2) + b(2)$	...
x_3	$ax_3^3 + bx_3^2 + cx_3 + d$	...	...	
x_4	$ax_4^3 + bx_4^2 + cx_4 + d$			

8. For the polynomial function $f(x) = 2x^5 + 123456789x^4$, the 5th order finite differences depend only on the leading term $2x^5$. Why?

x^4 term has 4th order finite differences constant. Therefore the 5th order finite differences coming from this term are all 0.

9. Given that the following relation may be modelled by a cubic function $y = f(x)$, find $f(-2)$ and $f(3)$.

x	y	Δ_1	Δ_2	Δ_3
-2	-12			
		10		
-1	-2		-8	
		2		6
0	0		-2	
		0		6
1	0		4	
		4		6
2	4		10	
		14		
3	18			

Δ_3 are constant

$f(3) = 18$

$f(-2) = -12$

10. Is the following relation linear? Justify your answer.

	x	y	
	-1	0	
3	2	6	6
1	3	12	6

x-intervals are not echidistant

No. m_s over $[-1,2]$ is $\frac{6-0}{2-(-1)} = 2$

m_s over $[2,3]$ is $\frac{12-6}{3-2} = 6$

For any linear relation the m_s should be constant

m_s is the slope of secant line

11. Is the following relation quadratic? Justify your answer.

x	y		
-4	~~16~~ 12		
		-10	
-2	2		8
		-2	
0	0		8
		6	
2	6		8
		14	
4	20		

echidistant x intervals

Yes. It is quadratic