

### 3.1 Exploring Polynomial Functions

#### Challenge Questions

1. Are the following functions polynomial? Justify your answer.

a)  $f(x) = x - (x^{1/3})^6$

b)  $f(x) = \sqrt{(x^2 + 1)^2}$

2. Find the leading term of the following polynomial function.

$$f(x) = -(1 - 2x)^3(3x - 1)^2(1 - x + 2x^2)$$

3. If the y-intercept of the polynomial function  $p(x)$  is  $-3$  and the y-intercept of the polynomial function  $q(x)$  is  $2$ , find the y-intercept of the function  $f(x) = (p^2(x) - 2q(x) + 1)^2$ .

4. If the degree of the polynomial function  $p(x)$  is  $2$  and the degree of the polynomial function  $q(x)$  is  $3$ , find the degree of the polynomial function  $f(x) = (p^2(x) - q^2(x))^2$ .

5. If the leading term of the polynomial function  $p(x)$  is  $-2x^2$  and the leading term of the polynomial function  $q(x)$  is  $x^3$ , find the leading term of the polynomial function  $f(x) = (p(x) + q(x))^2 - p^3(x)$ .

6. Find the leading term of the following polynomial function given by a table of values.

x	y
-3	80
-2	27
-1	4
0	-1
1	0
2	-5
3	-28

7. Prove that the 3<sup>rd</sup> order finite differences of any cubic polynomial function in the form  $f(x) = ax^3 + bx^2 + cx + d$  are constant and equal to  $(3 \times 2 \times 1)a$ .

8. For the polynomial function  $f(x) = 2x^5 + 123456789x^4$ , the 5<sup>th</sup> order finite differences depend only on the leading term  $2x^5$ . Why?

9. Given that the following relation may be modelled by a cubic function  $y = f(x)$ , find  $f(-2)$  and  $f(3)$ .

x	y
-1	-2
0	0
1	0
2	4

10. Is the following relation linear? Justify your answer.

x	y
-1	0
2	6
3	12

11. Is the following relation quadratic? Justify your answer.

x	y
-4	12
-2	2
0	0
2	6
4	20