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3.1 Exploring Polynomial Functions

A Polynomial Functions

A *polynomial function* $y = f(x)$ is defined by: $x^0 = 1$

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

$f(x) = -2x^3 + 4x^2 - 5x^0 \Rightarrow n = 3$

$a_n = -2$

where:

- $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ are *real numbers* called the *coefficients* of the polynomial function $-2, \frac{3}{4}, \sqrt{2}, \pi$
- a_n is called *leading coefficient*
- $\boxed{a_n x^n}$ is called *leading term* $LT = -2x^3$
- a_0 is called the *constant term* $a_0 = -5$
- n is a *non-negative integer* that gives the *degree* of the polynomial function

Note. The degree of the polynomial function n is the largest exponent of x

Ex 1. Verify if the following expressions are or not polynomial functions.

a) $f(x) = \sqrt{2}x^3 - 2x^2$ $n=3$ $a_0 = 0$

Yes

b) $f(x) = 2\sqrt{x} - x^2$ $\sqrt{x} = x^{1/2}$ $\frac{1}{2}$ is not integer

No

c) $f(x) = x^2 + \frac{1}{x}$ $\frac{1}{x} = x^{-1}$ -1 is not a positive integer

No

d) $f(x) = (x-1)(x+2)^2 = x^3 + \ldots - 4$

Yes

B Order

The terms of a polynomial function can be written in any order because the addition operation is a commutative operation.

Ex 2. Consider $f(x) = x - 2x^3 - 4x^2 + 3 - x^4$

a) Is this function polynomial? If yes, find the degree, the leading term, the leading coefficient, and the constant term

Yes $n = 4$

$LT = -x^4$ $LC = -1$

$CT = 3$

b) write the polynomial function in order of increasing powers of the variable x

$f(x) = 3 + x - 4x^2 - 2x^3 - x^4$

c) write the polynomial function in order of decreasing powers of the variable x

$f(x) = -x^4 - 2x^3 - 4x^2 + x + 3$

C Specific Polynomials

If $n = 0$, $f(x) = a_0$ is called *constant* function.
If $n = 1$, $f(x) = a_1 x + a_0$ is called *linear* function.
If $n = 2$, $f(x) = a_2 x^2 + a_1 x + a_0$ is called *quadratic* function.
If $n = 3$, $f(x) = a_3 x^3 + a_2 x^2 + a_1 x + a_0$ is called *cubic* function.

Note. For $n = 4$ we have the *quartic* function and for $n = 5$ we have the *quintic* function.

Ex 3. Identify each polynomial function as constant, linear, quadratic, cubic, quartic, or quintic.

a) $f(x) = -2$ constant

b) $f(x) = -x^2 + 3$ quadratic

c) $f(x) = 2x^3 - 3x^2 + x$ cubic

d) $f(x) = 2 - 3x$ linear

e) $f(x) = x^5 + x^3$ quintic

f) $f(x) = 1 - x^2 - x^4 + x$

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D Operations with polynomial functions

All the four operations (addition, subtraction, multiplication, and division) are defined for polynomial functions.

Ex 4. Consider two polynomial functions $f(x)=6x-3x^2$ and $g(x)=x-2$. Do the required operations:

a) $f(x)+g(x) = (6x-3x^2)+(x-2)$
$= -3x^2+7x-2$

b) $f(x)-g(x) = (6x-3x^2)-(x-2)$
$= -3x^2+5x+2$

c) $f(x)g(x) = (6x-3x^2)(x-2) = 6x^2-12x-3x^3+6x^2$
$= -3x^3+12x^2-12x$

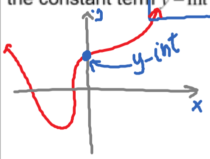
d) $f(x)/g(x) = \dfrac{6x-3x^2}{x-2} = \dfrac{3x(2-x)}{x-2} = -3x;\ x\neq 2$

E y-intercept

The y-intercept of a polynomial function is equal with the constant term $\boxed{y-\text{int}=f(0)=a_0}$

$x=0$

constant term



y-int point $= (0, f(0))$

y-int value $= f(0)$

Ex 5. Find the y-intercept for each polynomial function.

a) $f(x)=-2 \longrightarrow$ y-int $=-2$

b) $f(x)=-x^2+3 \longrightarrow 3$

c) $f(x)=2x^3-3x^2+x \longrightarrow 0$

d) $f(x)=(x^2+1)(x-2) \longrightarrow f(0)=(0^2+1)(0-2)=(1)(-2)=-2=$ y-int

e) $f(x)=(x^3-2)^3 \longrightarrow -8$

f) $f(x)=-2(x+3)^2(x-1)^5 \longrightarrow -2(9)(-1)=+18$

F Finite Differences

The *nth* finite differences of a polynomial function of degree n are constant.
This constant c is related to a_n and n by:

$$c=n!a_n$$

where $n!$ (n factorial) is defined by

$$n!=1\times 2\times 3\times\ldots\times(n-1)\times n$$

Note: Use "following # minus preceding #" rule to find the differences:

$$\begin{array}{ll} a & \\ & b-a \\ b & \\ & c-b \\ c & \end{array}$$

Ex 6. Use the information provided bellow and the finite differences method to find the *degree* of the polynomial function and the *leading coefficient*.

x	y	First $\Delta^1 y$	Second $\Delta^2 y$	Third $\Delta^3 y$	Fourth $\Delta^4 y$	$\Delta^5 y$
-4	-476					
-3	-134	342				
-2	-10	124	-218			
-1	16	26	-98	120		
0	16	0	-26	72	-48	
1	14	-2	-2	24	-48	
2	-14	-28	-26	-24	-48	
3	-140	-126	-98	-72	-48	
4	-484	-344	-218	-120	-48	

same number $= c$

$\therefore$ This relation is quartic
$n=4$

$c=(n!)a_n$

-48

$4! = 4(3)(2)(1)$
$=24$

$a_n = \dfrac{-48}{24} = -2$

optional

Reading: Nelson Textbook, Pages 124-126
Homework: Nelson Textbook, Page 127: #1, 2, 5

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