2.2 Estimating Instantaneous Rate of Change

A Instantaneous Rate of Change

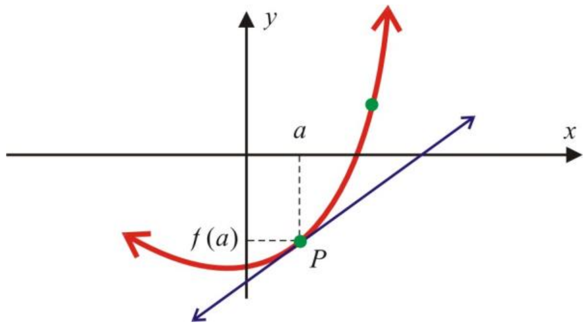
If x_1 and x_2 are approaching a number $x = a$ then the *Average Rate of Change* is called the *Instantaneous Rate of Change* at $x = a$.

$$IRC = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{x_1, x_2 \to a} \frac{y_2 - y_1}{x_2 - x_1} = \lim_{x_1, x_2 \to a} \frac{f(x_2) - f(x_1)}{x_2 - x_1}$$

Note: The *Instantaneous Rate of Change* is the same as the *Slope of the Tangent Line* at $P(a, f(a))$.

B Tangent Line

As the point Q approaches the point P, the secant line approaches the *tangent line* at P.

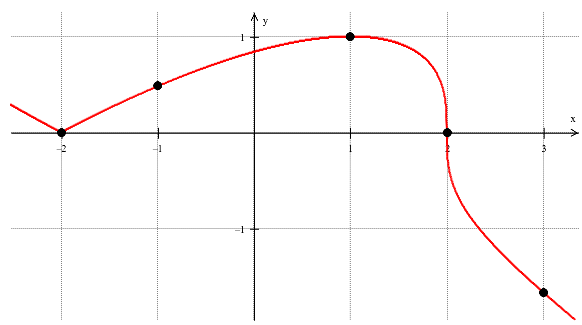


https://www.desmos.com/calculator/t21p9dcpgr

Ex 1. The function $y = f(x)$ is represented graphically on the right figure.

a) Draw the tangent line at each point emphasized on the graph.

b) For each point emphasized on the graph, find if the slope of the tangent line is negative, zero, positive, positive or negative infinity, or does not exist.



C Increasing/Decreasing Functions

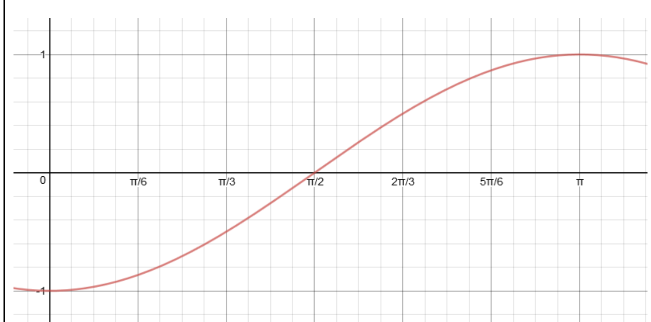
The function $y = f(x)$ is increasing at $x = a$ if

$$IRC = m_T > 0$$

and is decreasing if

$$IRC = m_T < 0$$

Ex 2. The graph of the function $y = \sin(x - \pi / 2)$ is given below.



a) What happens to the slope of the tangent line as x changes from $x = 0$ to $x = \pi$.

b) Estimate the maximum value of the slope the tangent line and specify the number x when this maximum value occurs.

D Preceding Intervals

If $x_1 = a - h$ and $x_2 = a$ then the IRC may be estimated by using the formula:

$$IRC \cong \frac{f(a) - f(a-h)}{h}$$

where h is a small number.

Note, The smaller is the number h, the more accurate is the IRC estimation.

E Following Intervals

If $x_1 = a$ and $x_2 = a + h$ then the IRC may be estimated by using the formula:

$$IRC \cong \frac{f(a+h) - f(a)}{h}$$

where h is a small number.

Note, The smaller is the number h, the more accurate is the IRC estimation.

F Symmetric Intervals

If $x_1 = a - h$ and $x_2 = a + h$ then the IRC may be estimated by using the formula:

$$IRC \cong \frac{f(a+h) - f(a-h)}{2h}$$

where h is a small number.

Note, The smaller is the number h, the more accurate is the IRC estimation.

Ex 3. Let $f(x) = 2x^3 + 1$. Estimate the instantaneous rate of change at $x = 1$ by using $h = 0.01$ and

a) A preceding interval

b) A following interval

c) A symmetric interval

G Algebraic Computation and the Exact Value

Use the formula

$$IRC = m_T = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

to find the exact value of the instantaneous rate of change at $x = a$ or the slope of the tangent line at $P(a, f(a))$.

Note. In this case h is not given and is replaced by 0 at the end of the computation process.

Ex 4. Find the exact value of the slope of the tangent line to the curve $y = f(x) = x^2 + x$ at the point $P(1,2)$.

Reading: Nelson Textbook, Pages 79-85
Homework: Nelson Textbook, Page 86: #2, 3, 5, 8