2.1 Determining Average Rate of Change

A Average Rate of Change

$y = f(x)$, $y_1 = f(x_1)$, $y_2 = f(x_2)$
$\Delta x = x_2 - x_1$ (change in variable x)
$\Delta y = y_2 - y_1$ (change in variable y)

The *Average Rate of Change* (ARC) in the y variable with respect to the x variable, on (over) the interval $[x_1, x_2]$ (or $x_1 \le x \le x_2$) is given by:

$$ARC = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = m_S$$

Note. The unit of ARC is:

$$unit(ARC) = \frac{unit(\Delta y)}{unit(\Delta x)}$$

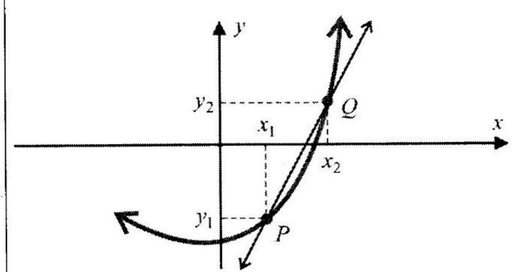
Note: The *Average Rate of Change* (ARC) is equal to the *slope of the secant line* (m_S) passing through the points $P(x_1, y_1)$ and $Q(x_2, y_2)$.

B Secant Line

Let $y = f(x)$ be a function and $P(x_1, y_1)$ and $Q(x_2, y_2)$ two points on its graph.

The *slope* of the *secant line* (m_S) that passes through the points P and Q is given by:

$$m_S = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = ARC$$



Ex 1. A rock is launched vertically upward. The height h (in meters) at the time t (in seconds) of the rock is given by $h(t) = 100t - 10t^2$. Find the average velocity (ARC) over the third second of motion.

0 1 2 3 4 t

third second

$t_1 = 2$
$h_1 = 100(2) - 10(2^2) = 160$

$t_2 = 3$
$h_2 = 100(3) - 10(3^2) = 210$

$$AV = \frac{h_2 - h_1}{t_2 - t_1} = \frac{210 - 160}{3 - 2} = 50$$

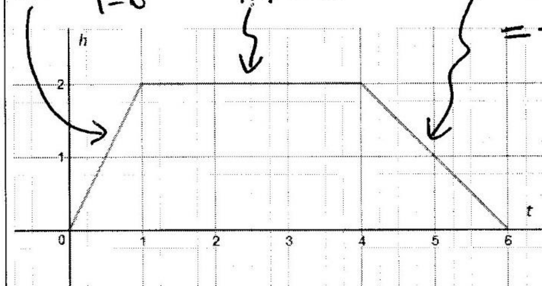
∴ The average velocity over the third second is 50 m/s

Ex 2. In the figure below is represented the position h (in kilometers) at the time t (in hours) of a baloon. Describe the motion of the ballon in terms of average velocity.

$AV = \frac{2-0}{1-0}$ // 2 km/h

$AV = 0$

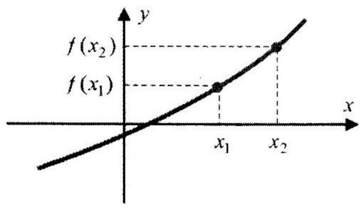
$AV = \frac{0-2}{6-4} = -1$ km/h



∴ The balloon is moving up at 2 km/h for one hour, then is not moving for 3 hours and finally is moving down at 1 km/h for two hours.

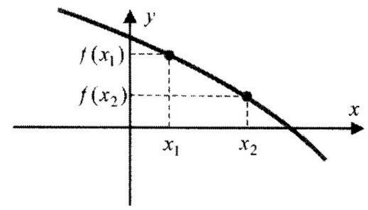
C Increasing Functions

A function f is *increasing* over the interval (a,b) if $ARC = m_S = \frac{f(x_2)-f(x_1)}{x_2-x_1} > 0$ for all x_1, x_2 in the interval (a,b).



D Decreasing Functions

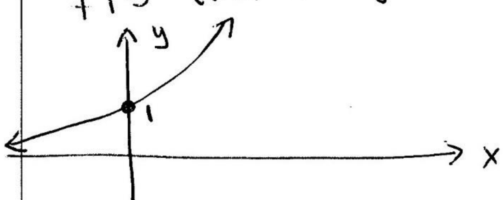
A function f is *decreasing* over the interval (a,b) if $ARC = m_S = \frac{f(x_2)-f(x_1)}{x_2-x_1} < 0$ for all x_1, x_2 in the interval (a,b).



Ex 3. Prove that the function $y = f(x) = 10^x$ is increasing over its domain.

$b = 10 > 1$

f is increasing



Ex 4. Prove that the average rate of change is constant for a linear function.

$\to f(x) = ax + b = y$

$y_1 = ax_1 + b$

$y_2 = ax_2 + b$

$y_2 - y_1 = a(x_2 - x_1)$

$ARC = \frac{y_2 - y_1}{x_2 - x_1} = a =$ constant

Ex 5. During an experiment the number of bacteria is measured every minutes (for ten minutes) and the results are presented below:

t	N
0	100
1	200
2	400
3	800
4	1600
5	3200
6	6400
7	12800
8	25600
9	51200
10	102400

first two minutes

last two minutes

$ARC_1 = \frac{400-100}{2-0}$

$= 150$ bacteria/minute

$ARC_2 = \frac{102400-25600}{10-8}$

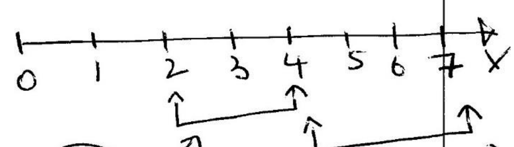
$= 38400 \frac{\text{bacteria}}{\text{minute}}$

$ARC_2 \gg ARC_1$

much more greater

Compare the average rate of change during the first two minutes and the average rate of change during the last two minutes of the experiment.

Ex 6. For a given function, the average rate of change over [2,4] is 5 and the average rate of change over [4,7] is −2. Find the average rate of change over [2,7].



$\Delta y = 5(2)$
$= 10$

$\Delta y = (-2)(3)$
$= -6$

$ARC = \frac{10 + (-6)}{2+3} = \frac{4}{5}$

$\therefore$ ARC is $4/5$

Reading: Nelson Textbook, Pages 68-75
Homework: Nelson Textbook, Page 76: #4, 8, 10