### 2.1 Determining Average Rate of Change

#### A Average Rate of Change

Given a function \( y = f(x) \), let \( y_1 = f(x_1) \) and \( y_2 = f(x_2) \). The average rate of change \((ARC)\) in the \( y \) variable with respect to the \( x \) variable, on the interval \([x_1, x_2]\) (or \( x_1 \leq x \leq x_2 \)) is given by:

\[
ARC = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = m_S
\]

The unit of \( ARC \) is:

\[
\text{unit}(ARC) = \frac{\text{unit}(\Delta y)}{\text{unit}(\Delta x)}
\]

#### B Secant Line

Let \( y = f(x) \) be a function and \( P(x_1, y_1) \) and \( Q(x_2, y_2) \) two points on its graph.

The slope of the secant line \( (m_S) \) that passes through the points \( P \) and \( Q \) is given by:

\[
m_S = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = f(x_2) - f(x_1) = \Delta y
\]

**Ex 1.** A rock is launched vertically upward. The height \( h \) (in meters) at the time \( t \) (in seconds) of the rock is given by \( h(t) = 100t - 10t^2 \). Find the average velocity \((ARC)\) over the third second of motion.

**Ex 2.** In the figure below is represented the position \( h \) (in kilometers) at the time \( t \) (in hours) of a balloon. Describe the motion of the balloon in terms of average velocity.
### C Increasing Functions

A function \( f \) is *increasing* over the interval \((a, b)\) if
\[
ARC = m_S = \frac{f(x_2) - f(x_1)}{x_2 - x_1} > 0 \text{ for all } x_1, x_2 \text{ in the interval } (a, b).
\]

### D Decreasing Functions

A function \( f \) is *decreasing* over the interval \((a, b)\) if
\[
ARC = m_S = \frac{f(x_2) - f(x_1)}{x_2 - x_1} < 0 \text{ for all } x_1, x_2 \text{ in the interval } (a, b).
\]

### Ex 3.
Prove that the function \( y = f(x) = 10^x \) is increasing over its domain.

### Ex 4.
Prove that the average rate of change is constant for a linear function.

### Ex 5.
During an experiment the number of bacteria is measured every minutes (for ten minutes) and the results are presented below:

<table>
<thead>
<tr>
<th>( t )</th>
<th>( N )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>1</td>
<td>200</td>
</tr>
<tr>
<td>2</td>
<td>400</td>
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<td>6</td>
<td>6400</td>
</tr>
<tr>
<td>7</td>
<td>12800</td>
</tr>
<tr>
<td>8</td>
<td>25600</td>
</tr>
<tr>
<td>9</td>
<td>51200</td>
</tr>
<tr>
<td>10</td>
<td>102400</td>
</tr>
</tbody>
</table>

Compare the average rate of change during the first two minutes and the average rate of change during the last two minutes of the experiment.

### Ex 6.
For a given function, the average rate of change over \([2, 4]\) is 5 and the average rate of change over \([4, 7]\) is -2. Find the average rate of change over \([2, 7]\).

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**Reading:** Nelson Textbook, Pages 68-75  
**Homework:** Nelson Textbook, Page 76: #4, 8, 10