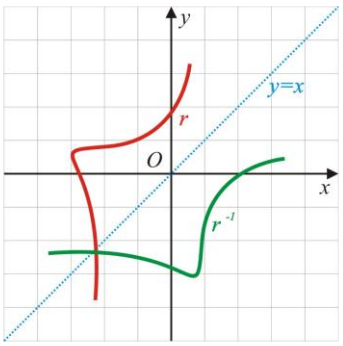
1.5 Inverse Relations

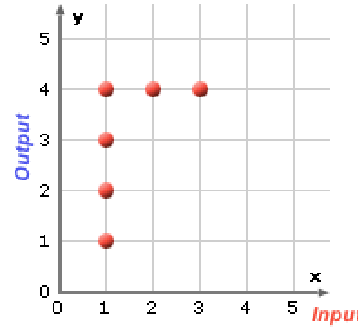
A Inverse Relation
For any relation there is an inverse relation obtained by interchanging (switching) x and y for all the elements (ordered pairs) of the original relation.
The inverse relation of the relation r is denoted by r^{-1}.

Ex 1. Find the inverse relation of the relation $r = \{(1,2),(1,0),(-2,1),(0,2)\}$

B Symmetry
The graph of a relation and the graph of its inverse relation are symmetrical about the line $y = x$.



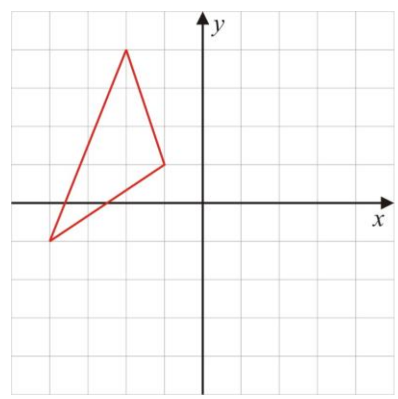
Ex 2. A relation r is given by the following graph. Find and graph the inverse relation r^{-1} and observe the symmetry.



C Corresponding Key Points
A point $P(x, y)$ on the relation r corresponds to the point $P'(y, x)$ on the inverse relation r^{-1}.

The points P and P' are symmetrical about the line $y = x$.

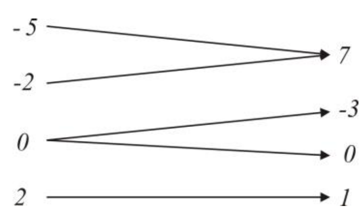
Ex 3. A relation is given by the graph to the right. Use corresponding key points to graph the inverse relation.



D Domain and Range
The domain of the inverse relation r^{-1} is the same as the range of the relation r: $D_{r^{-1}} = R_r$.

The range of the inverse relation r^{-1} is the same as the domain of the relation r: $R_{r^{-1}} = D_r$.

Ex 4. A relation r is given by the following mapping diagram:



a) Find the domain and the range of the relation r.

b) Find the domain and the range of the relation r^{-1}.

c) Explain how you would get the inverse relation.

E Inverse Relation of a Function
Any function is a relation.
So, any function f has an inverse relation f^{-1}.

Note: The inverse relation of a function may be or not a function.

Ex 5. For each case, use key points to graph the function and its inverse relation. Is the inverse relation a function?

a) $y = x^2$

b) $y = |x - 1|$

c) $y = 2 - \sqrt{x-1}$

F Algebraic Method
To find the inverse of a function:
a) write the original function in the form $y = f(x)$
b) switch the variable x and y
c) solve the last expression for y
d) replace y by $f^{-1}(x)$

Ex 6. Find the inverse of each one-to-one function. State the domain and the range for the function and the inverse function.

a) $f(x) = -2x + 3$

b) $f(x) = \dfrac{x-1}{x+2}$

c) $f(x) = 1 - 2\sqrt{x-3}$

*d) $y = x + \sqrt{x}$

G One-to-One Functions
If the inverse relation of a given function f is also a function, then the original function f is called one-to-one function. In this case:

$$y = f(x) \Leftrightarrow x = f^{-1}(y)$$

Ex 7. Prove that the following relations are true for any one-to-one function $f : X \to Y$.

a) $f(f^{-1}(x)) = x \quad \text{for any } x \in Y$

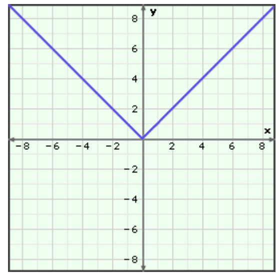
b) $f^{-1}(f(x)) = x \quad \text{for any } x \in X$

H Horizontal Line Test
One-to-one functions pass the horizontal line test:

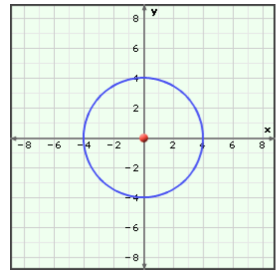
Any horizontal line intersects the graph in at most one point.

Ex 8. Classify each graph as a relation, function or one-to-one function.

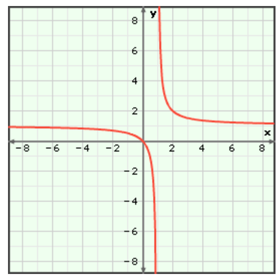
a)



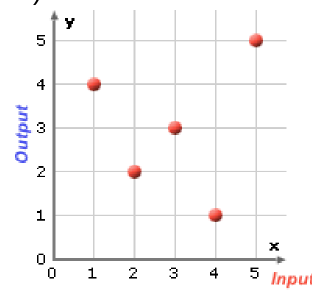
b)



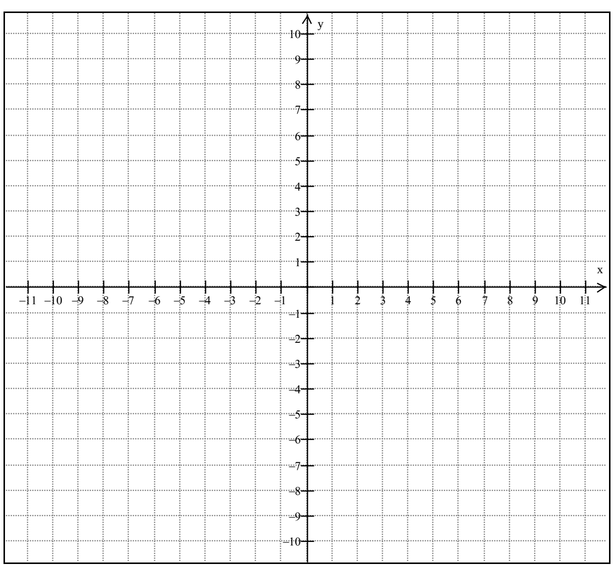
c)



d)



I Restricted Domains
By restricting the domain of a function (which is not one-to-one), we may obtain a one-to-one function.



Ex 9. Consider the function:
$f(x) = -(x-1)(x+5) \quad , \quad x \le -2$

a) Convert the function to the vertex form.

b) State the domain and the range of the function $f(x)$.

c) Find the inverse function $f^{-1}(x)$.

d) State the domain and the range of the function $f^{-1}(x)$.

e) Sketch the graph of the functions $f(x)$ and $f^{-1}(x)$ on the grid provided to the left.

Reading: Nelson Textbook, Pages 38-43
Homework: Nelson Textbook, Page 43: #1af, 2cd, 3, 4, 5, 6, 7, 9, 10, 12, 13, 15, 17