1.4 Sketching Graphs of Functions

A Parent Functions

The parent functions are functions in simplest form. We may use parent functions and transformations to create more complicated functions.

For example, the function $g(x) = -2\dfrac{1}{(x-1)^2} + 3$ is a transformation of the parent function $f(x) = \dfrac{1}{x^2}$.

To graph a parent function use key points.

For example, five key points of the parent function $f(x) = \sqrt[3]{x}$ are $(-8,-2)$, $(-1,-1)$, $(0,0)$, $(1,1)$, and $(8,2)$.

Ex 1. Find five key points to graph each of the following parent functions:

a) $f(x) = x$ (linear function)
b) $f(x) = x^2$ (quadratic function)
c) $f(x) = x^3$ (cubic function)
d) $f(x) = x^4$ (quartic function)
e) $f(x) = \dfrac{1}{x}$ (reciprocal function)
f) $f(x) = \dfrac{1}{x^2}$
g) $f(x) = \sqrt{x}$ (square root function)
h) $f(x) = \sqrt[3]{x}$ (cube root function)
i) $f(x) = x^{2/3}$
j) $f(x) = x^{4/3}$
k) $f(x) = \dfrac{1}{x^{2/3}}$
l) $f(x) = |x|$ (absolute value function)
m) $f(x) = 2^x$ (exponential function)
n) $f(x) = \sin(x)$ (sine function)
o) $f(x) = \cos(x)$ (cosine function)

B Transformations

Given a parent function f, we can create new functions using transformations:

$$g(x) = af(b(x-c)) + d$$

If $|a| > 1$, graph is vertically stretched by a factor of $|a|$.
If $|a| < 1$, graph is vertical compressed by a factor of $|a|$.
If $a < 0$, graph is reflected about the x axis.

If $|b| > 1$, graph is horizontally compressed by a factor of $1/|b|$.
If $|b| < 1$, graph is horizontally stretched by a factor of $1/|b|$.
If $b < 0$, graph is reflected about the y axis.

If $c \neq 0$, graph is horizontally translated (shifted) to the right (if $c > 0$) or to the left (if $c < 0$).

If $d \neq 0$, graph is vertically translated (shifted) upward (if $d > 0$) or downward (if $d < 0$).

Ex 2. For each case, use transformations to graph.

a) $f(x) = x + 2$
b) $f(x) = 1 - (x-2)^2$
c) $f(x) = (x+1)^3 + 2$
d) $f(x) = -(x-2)^4$
e) $f(x) = \dfrac{-2}{x+3} - 1$
f) $f(x) = -\dfrac{1}{(x-1)^2} + 3$
g) $f(x) = 2 - \sqrt{1-x}$
h) $f(x) = 2 + \sqrt[3]{x-1}$
i) $f(x) = 1 + (x-1)^{2/3}$
j) $f(x) = (x+2)^{4/3}$
k) $f(x) = \dfrac{-1}{(x-1)^{2/3}}$
l) $f(x) = 3 - |1-2x|$
m) $f(x) = 1 - 2^{x+3}$
n) $f(x) = -2\sin(x - 45°)$
o) $f(x) = 1 + 2\cos(2x)$

C Mapping Formulas

By comparing the original (parent or old) function:

$$y_{old} = f(x_{old})$$

and the image (new) function:

$$y_{new} = af[b(x_{new} - c)] + d$$

we get:

$$\begin{cases} y_{new} = ay_{old} + d \\ x_{old} = b(x_{new} - c) \end{cases}$$

or:

$$\begin{cases} y_{new} = ay_{old} + d \\ x_{new} = \dfrac{x_{old}}{b} + c \end{cases} \qquad (*)$$

A point (x_{old}, y_{old}) on the original (parent or old) function corresponds to the point (x_{new}, y_{new}) on the image (new) function.

Note: According to the mapping formula (*) and order of operations:

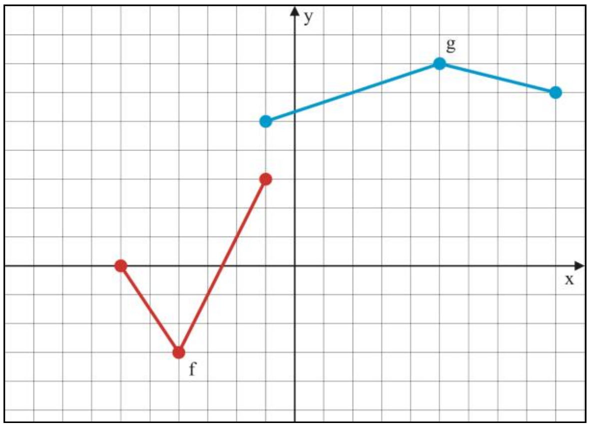
- Vertical stretch/compression/reflection (a) must be done before the vertical translation (d).
- Horizontal stretch/compression/reflection (b) must be done before the horizontal translation (c).

Ex 3. Graph each function by using a few key points and the mapping formulas.

a) $f(x) = -2|1 - x| + 3$

b) $f(x) = 3 - 2\sqrt{4 - x}$

Ex 4. Write the function $g(x)$ in the form: $g(x) = af[b(x - c)] + d$.



D Domain and Range

After transformations, the domain and the range may be changed. Use the mapping formulas (*) to find the new ones.

Ex 4. A function f with a domain $D = (-1, 3]$ and a range $R = [2, \infty)$ is transformed into a new function g by $g(x) = -2f(2x - 3) + 4$. Find the domain and the range of the new function g .

Reading: Nelson Textbook, Pages 29-35

Homework: Nelson Textbook, Page 35: #1cf, 2, 3, 4d, 5, 7, 8, 9cf, 10, 15

