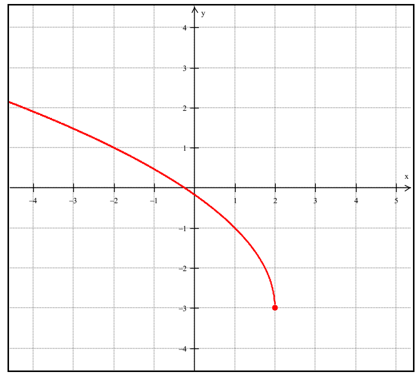
1.3 Properties of Graphs of Functions

A Domain and Range

The domain of a function is the set of all x values where the function is defined.
The range of a function is the set of all y values such that $y = f(x)$.

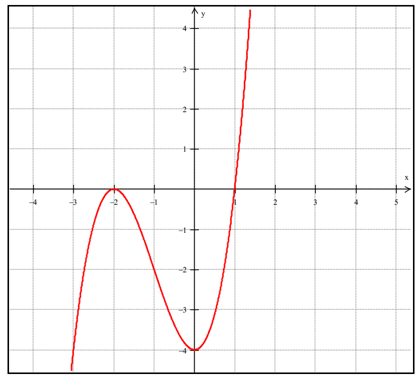
Ex 1. Find the domain and the range for the function represented graphically to the right.



B x-intercepts and y-intercept

The x-intercepts are the $x-\text{int}$ values such that $f(x-\text{int}) = 0$.
The y-intercept is the $y-\text{int}$ value such that $y-\text{int} = f(0)$ (if it exists).

Ex 2. Find the x- and the y-intercepts for the function represented graphically to the right.



C Intervals of Increase or Decrease (Turning Points)

The function increases if the slope of the tangent line is positive (the graph is going right and up).
The function decreases if the slope of the tangent line is negative (the graph is going right and down).
A turning point is a point where the function changes from increasing to decreasing or vice versa.

Ex 3. Find the intervals of increase/decrease and the turning points for the function given at example 2.

D Maximum and Minimum Points

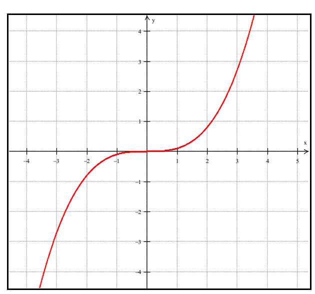
The point $(a, f(a))$ is a maximum point if $f(a) \geq f(x)$ in a neighborhood of $x = a$.
The point $(a, f(a))$ is a minimum point if $f(a) \leq f(x)$ in a neighborhood of $x = a$.

Ex 4. Find the (local) maximum and minimum points for the function given at example 2.

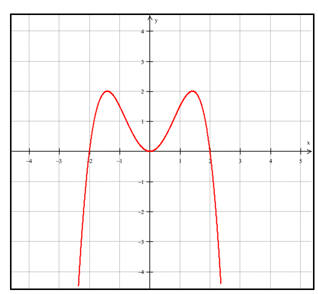
E Odd and Even Functions

The function f is even if $f(-x) = f(x)$ (the graph is symmetric about the y-axis).
The function f is odd if $f(-x) = -f(x)$ (the graph is symmetric about the origin).

Ex 5. Classify as odd or even function.



a)

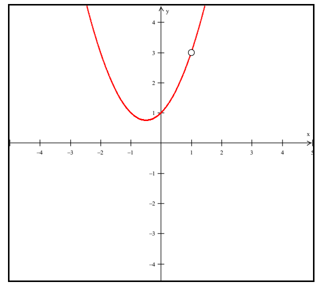


b)

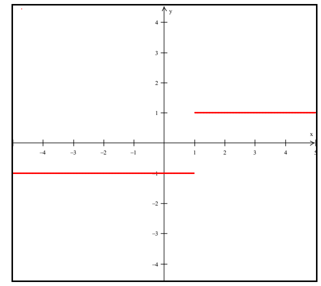
F Continuous and Discontinuous Functions
The graph of a continuous function can be drawn "without lifting pencil from paper".

A continuous function has no holes, finite gaps (jumps), or infinite breaks.

Ex 6. Identify holes, jumps, or infinite breaks for the functions represented to the right.



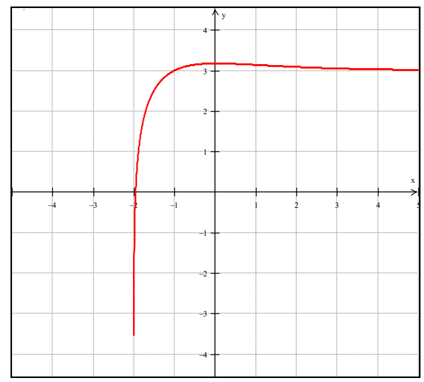
a)



b)

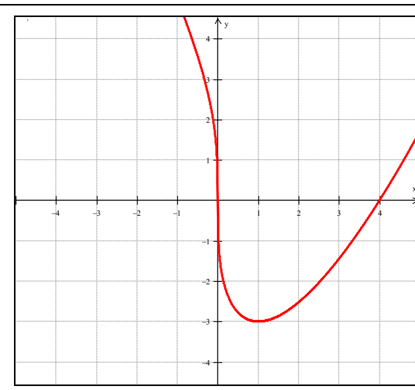
G Vertical and Horizontal Asymptotes
The vertical line $x=a$ is a vertical asymptotes if the y values become unbounded (approaches infinity) in the neighborhood of $x=a$.
The horizontal line $y=a$ is a horizontal asymptotes if the values of y become close to a as x becomes unbounded (approaches infinity).

Ex 7. Find the equation of any vertical or horizontal asymptote (if exists) for the function represented graphically to the right.



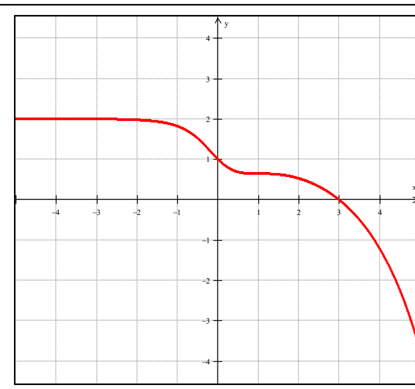
H Horizontal and Vertical Tangent Lines
The graph of a function may have points where the tangent line is horizontal (slope is zero) or vertical (the slope is unbounded (approaches infinity)).

Ex 8. Find points on the graph of the function, represented graphically to the right, where the tangent line is horizontal or vertical.



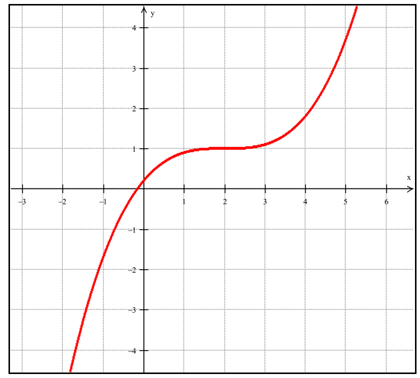
I End Behaviour
The end behaviour is related to the y values as x becomes unbounded (approaches infinity).

Ex 9. Find the end behaviour of the the function represented graphically to the right.



J Concavity Upward and Downward
The graph of a function has a concavity upward if the graph lies above all its tangents.
The graph of a function has a concavity downward if the graph lies below all its tangents.

Ex 10. Find the intervals where the graph is concave upward or downward for the function represented graphically to the right.

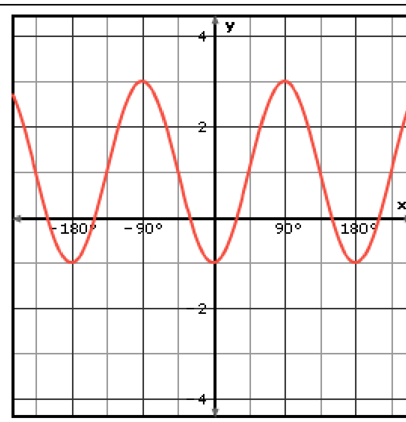


K Corner, Cusp and Infinite Slope Points
A corner point is a point with two distinct tangent lines.
A cusp point is a turning point with a vertical tangent line.
An infinite slope point is a non turning point with a vertical tangent line.

Ex 11. Find the corner, cusp or infinite slope points on the graph of the function represented graphically to the right.

L Periodic Functions
A function is periodic if there exists T such that $f(x+T)=f(x)$.

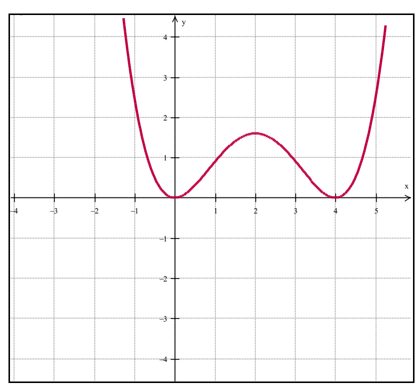
Ex 12. Find the period of the function represented graphically to the right.



M Axis of Symmetry and Axis
The graph of a function has an axis of symmetry $x=a$ if the graph is symmetric about the vertical line $x=a$.
The sine and cosine functions have an axis defined by: $y=(y_{\max}+y_{\min})/2$.

Ex 13. Find the equation of the axis for the function at example 11.

Ex 14. Find the equation of the axis of symmetry for the following function.



Ex 15. Graph (using a table of values or technology) and describe the properties of each function.

a) $f(x) = x$ (linear function)

b) $f(x) = x^2$ (quadratic function)

c) $f(x) = x^3$ (cubic function)

d) $f(x) = x^4$ (quartic function)

e) $f(x) = \frac{1}{x}$ (reciprocal function)

f) $f(x) = \frac{1}{x^2}$

g) $f(x) = \sqrt{x}$ (square root function)

h) $f(x) = \sqrt[3]{x}$ (cube root function)

i) $f(x) = x^{2/3}$

j) $f(x) = \frac{1}{x^{2/3}}$

k) $f(x) = |x|$ (absolute value function)

l) $f(x) = 2^x$ (exponential function)

m) $f(x) = \sin(x)$ (sine function)

Ex 16. Describe the graph of the power function

$f(x) = x^{\alpha}$; $x > 0, \alpha \in R$.

Ex 17. Describe the graph of the power function

$f(x) = x^{m/n}$; $m, n \in Z$.

Reading: Nelson Textbook, Pages 17-23
Homework: Nelson Textbook, Page 23: #4, 5, 7, 9, 11, 12, 14, 16

