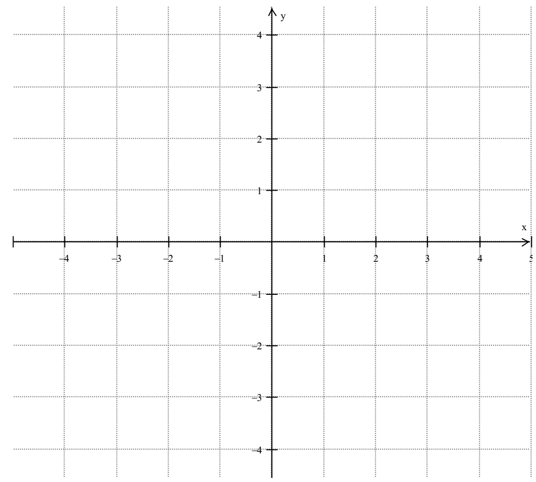
1.2 Exploring Absolute Value

A Absolute Value The absolute value $\|x\|$ of a real number x is the distance between that number and the number 0.	Ex 1. Evaluate the following expressions: a) $\|5\|$ b) $\|-5\|$ c) $\|0\|$ d) $\|+3\|$ e) $\|5-3\|$ f) $\|2-\|-3\|\|$ g) $\|1-2\|-\|2-1\|$ h) $\|\|-7\|-\|-2\|\|$
B Definition of Absolute Value The absolute value $\|x\|$ is defined by: $\|x\|=\begin{cases} x, & if\ \ x \ge 0 \\ -x, & if\ \ x < 0 \end{cases}$	Ex 2. Rewrite the following algebraic expression without the absolute value symbol **| |**. $\|x-3\|=$
C Properties of Absolute Value The absolute value has the following properties: a) $\|a\|=\|-a\|$ b) $\|a\|=0 \Leftrightarrow a=0$ c) $\|ab\|=\|a\|\|b\|$ d) $\left\|\frac{a}{b}\right\|=\frac{\|a\|}{\|b\|}$ e) $\|a+b\|\le\|a\|+\|b\|$ (triangle inequality)	Ex 3. Use the properties of the absolute value to simplify: a) $\frac{\|-2x\|}{\|-x\|}$ b) $\|x\|-\|-x\|$ c) $\left\|\frac{-2x}{3y}\right\|\left\|\frac{-2y}{3x}\right\|$ d) $\|-3x\|-\|-x\|-\|x\|$
D Distance between two numbers If $A(a)$ and $B(b)$ are two points on the number line corresponding to the numbers a and b respectively, the distance between the points can be expressed using the absolute value as: $d(A,B)=\|b-a\|$.	Ex 4. Solve for x. $\|x-3\|=\|5-x\|$
E Equations Consider $E(x)$ an algebraic expression containing the variable x. The equation $\|E(x)\|=a;\quad a\ge 0$ can be solved by isolating x from the equation $E(x)=\pm a$.	Ex 5. For each case, solve for x. a) $\|x\|=3$ b) $\|2x-1\|=3$ c) $\left\|2-\frac{2x+1}{2x-1}\right\|=1$

F Absolute Value Function
The absolute value function is defined by:

$$y = f(x) = |x|$$

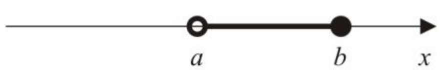
Ex 6. Graph the absolute function $y = f(x) = |x|$ and describe its properties (symmetry, domain and range).



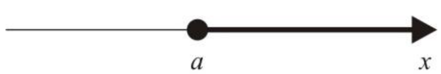
G Inequalities
The comparison operators are: < (less), $\leq$ (less or equal to), = (equal to), $\neq$ (not equal to), > (greater than), and $\geq$ (greater or equal to).
The comparison operators < (less), $\leq$ (less or equal to), > (greater than), and $\geq$ (greater or equal to) are used to create inequalities.

Ex 7. For each case, find the logical value (true or false) of the statement.
a) $-1 < 0$
b) $2 \leq 2$
c) $2 = 0$
d) $-1 \neq 1$
e) $-3 > 0$
f) $2 \geq -2$

H Interval Notation
The following notations are equivalent and represent sets of numbers:
$a < x \leq b$ (inequality notation)
$x \in [a, b)$ (interval notation)
$\{x \in R \mid a < x \leq b\}$ (set notation)

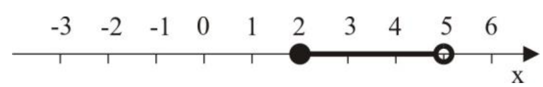


Similarly:
$x \geq a \Leftrightarrow x \in [a, \infty) \Leftrightarrow \{x \in R \mid x \geq a\}$

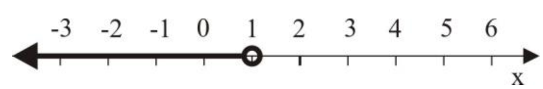


Ex 8. Write the following sets of numbers given graphically using various notations.

a)



b)



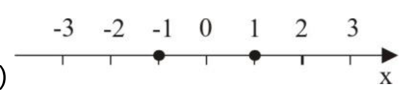
Ex 9. For each case, graph the solution set.

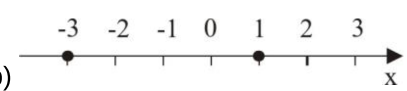
a) $|x| = 2$

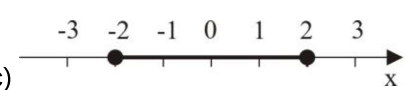
b) $|x - 2| = 3$

c) $|x| < 2$

Ex 10. Rewrite using the absolute value notation.

a)


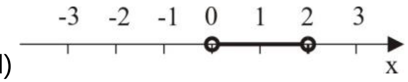
b)


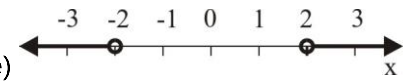
c)


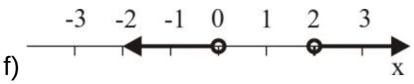
d) $|x-3| \le 2$

e) $|x| \ge 2$

f) $|2-x| \ge 3$

d) 

e) 

f) 

I Transformations

Given a parent function f, we can create new functions using transformations:

$$g(x) = af(b(x-c)) + d$$

If $|a| > 1$, there is a vertical stretch by a factor of $|a|$.
If $|a| < 1$, there is a vertical compression by a factor of $|a|$.
If $a < 0$, there is a reflection in the x axis.

If $|b| > 1$, there is a horizontal compression by a factor of $1/|b|$.
If $|b| < 1$, there is a horizontal stretch by a factor of $1/|b|$.
If $b < 0$, there is a reflection in the y axis.

If $c \neq 0$, there is a horizontal translation (shift) to the right (if $c > 0$) or to the left (if $c < 0$).

If $d \neq 0$, there is a vertical translation (shift) upward (if $d > 0$) or downward (if $d < 0$).

Ex 11. For each case, use transformations to graph.

a) $y = |x-3|$

b) $y = |x| + 2$

c) $y = |x+2| - 3$

d) $y = -2|3-x|$

e) $y = 4 - |3-2x|$

Reading: Nelson Textbook, Pages 14-15
Homework: Nelson Textbook, Page 16: #1-10

