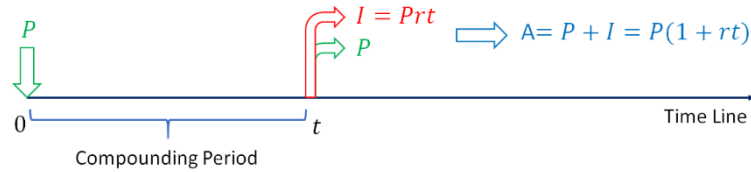


8.2 Compound Interest

A Simple Interest

- ✓ One time investment
- ✓ Interest is calculated only once, at the end of the compounding period (regularly one year)



where:

P is the *principal* or the *present value*

A is the *amount* or the *future value*

I is the total accumulated *interest*

r is the *interest rate* per compounding period (regularly given as percentage per year)

B Investigate a Simple Interest Investment

Example 1. For an investment of \$10,000 at 5% interest rate per year, complete the following table of values:

Present value or Principal $P = \$10,000$
 Interest Rate $r = 5\% = \frac{5}{100} = 0.05$

Time t (in years)	Accumulated Interest $I = Prt$	Future Value $A = P + I$
0 (now)	0	\$10,000
1 $t=1$	$I = (10,000)(0.05)(1)$ $= \$500$	$10,000 + \$500$ $= \$10,500$
2 $t=2$	$I = (10,000)(0.05)(2)$ $= \$1,000$	$10,000 + 1,000$ $= \$11,000$
3 $t=3$	$I = (10,000)(0.05)(3)$ $= \$1,500$	$10,000 + 1,500$ $= \$11,500$
4 $t=4$	$I = (10,000)(0.05)(4)$ $= \$2,000$	$10,000 + 2,000$ $= \$12,000$

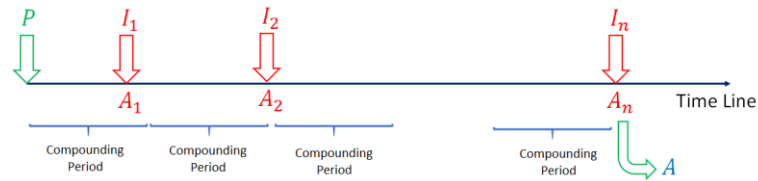
Use Desmos and make a Scatter Plot with the columns t and A .

What kind of relation is between A and t ?

∴ There is a linear relation between A and t

B Compound Interest

- ✓ Time Line is divided into *compounding periods*
- ✓ Interest is calculated *at the end of each compounding period and added to the principal*
- ✓ Interest rate is calculated per compounding period



C Interest Rate per Compounding Periods

If the interest rate r is given per year, then in order to find the interest rate per compounding period i , divide the annual interest rate r by the number of compounding periods in a year m :

$$i = \frac{r}{m}$$

Example 2. Complete the following table if the interest rate per year is 6% and the investment is over 3 years and 6 months

$$r = 6\% = \frac{6}{100} = 0.06$$

$$t = 3 \text{ years } 6 \text{ months} = 3.5 \text{ years}$$

Interest is compounded	Number of compounding periods per year m	Interest Rate per compounding period $i = \frac{r}{m}$	Number of compounding periods in 3 years and 6 months $n = m \cdot t$
annually	1	0.06 / 1	3.5
quarterly	4	0.06 / 4	4 * 3.5 = 14
monthly	12	0.06 / 12	12 * 3.5 = 42
weekly	52	0.06 / 52	52 * 3.5 = 182
daily	365	0.06 / 365	365 * 3.5 = 1277.5
bi-weekly	26	0.06 / 26	26 * 3.5 = 91

D Investigate a Compound Interest Investment

Example 3. For an investment of \$10,000 at 6% interest rate per year compounded monthly, complete the following table of values:

Present value or Principal $P =$

Interest Rate per Year $r =$

Interest Rate per compounding Period $i =$

Time t (in months)	Present Value at the beginning of the Compounding Period P	Accumulated Interest over the compounding period $I = Pi$	Future Value at the end of the Compounding Period $A = P + I$
0			
1			
2			
3			
4			

Use Desmos and make a Scatter Plot with the columns t and A .

What kind of relation is between A and t ?

E Compound Interest Formula

If an investment has a compounded interest, the future value is given by:

Handwritten notes and formulas:

- ① $t = ?$ (years)
- ② $m = ?$ (use the table)
- ③ $A = P(1+i)^n$
- ④ $i = \frac{r}{m}$
- ⑤ $A = P + I$
- ⑥ $n = m \cdot t$

where:
 P is the *principal* or the *present value*
 A is the *amount* or the *future value*
 I is the total accumulated *interest*
 r is the *interest rate* per year
 i is the *interest rate* per compounding period
 n is the *total number of compounding periods*

Example 4. Let consider an investment of \$25,000 with an interest rate of 5% compounded ~~quarterly~~ ^{monthly} over 4 years.

a) How many months are in one year?

$$m = 12$$

b) What is the interest rate per month?

$$i = \frac{r}{m} = \frac{0.05}{12}$$

c) How many months are in 4 years?

$$n = m \cdot t = (12)(4) = 48$$

d) What if the future value of the investment?

$$A = P(1+i)^n = \$25,000 \left(1 + \frac{0.05}{12}\right)^{48}$$

e) What is the accumulated interest over the 4 years period?

Example 5. Let consider buying a car of \$50,000 with a credit card with an interest rate of 20% compounded daily over 3 years.

a) How many days are in one year?

b) What is the interest rate per day?

c) How many days are in 3 years?

d) What if the future value of your credit card?

e) What is the accumulated interest over the 3 years period?

E Technology

We may use technology to find the answers for simple and compounded interest applications.

Free online Simple and Compound Interest calculator -> [Good Calculators](#)

Example 6. Use the online application to find the future value of an investment of \$12,000 at 3% interest rate per year compounded weekly over 5 years.

Reading Pages 430-432

Homework Pages 432-435 # 3, 5, 12