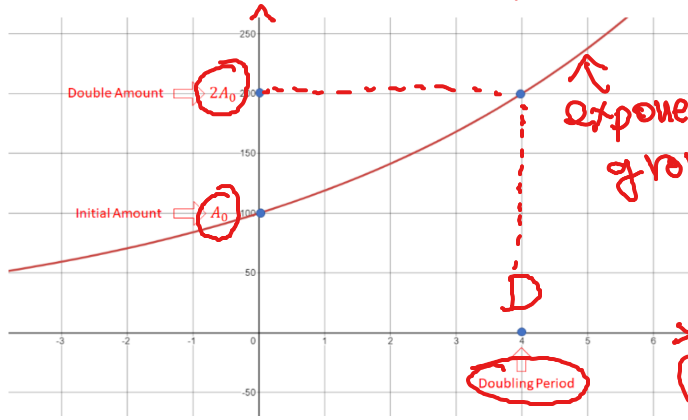
7.6 Solve Problems Involving Exponential Growth and Decay

A Exponential Growth and Doubling Period

A (Amount)

Exponential growth may be modelled by a formula

$$\boxed{A = A_0 2^{\frac{t}{D}}}$$



exponential growth

D

t (time)

where

- ✓ 2 is the base
- ✓ D is called doubling period
- ✓ t is time
- ✓ A_0 is called the initial amount
- ✓ A is the amount at time t

Example 1. The price of a house may be modeled by $P(t) = (\$150{,}000)\left(2^{\frac{t}{10}}\right)$, where t is the number of years after 2010.

a) What was the price of the house in 2010? $t = 0$ $2^{\frac{0}{10}} = 2^0 = 1$

∴ The price of the house in 2010 was $150,000 (initial amount)

b) What was the price of the house in the year of 2000 when it was build?

$t = -10$ (before)

$P(-10) = \$150{,}000\left(2^{\frac{-10}{10}}\right) = \frac{\$150{,}000}{2} = \$75{,}000$

∴ The price in 2000 was $75,000

c) What will the price be in 2030?

$t = 2030 - 2010 = 20$

$P(20) = \$150{,}000\left(2^{\frac{20}{10}}\right) = (\$150{,}000)(4) = \$600{,}000$

∴ The price in 2030 will be around $600,000

d) In what year will the price be double? $t = ?$ when $P(t) = 2\cdot(\$150{,}000)$

$2\cdot(\cancel{\$150{,}000}) = (\cancel{\$150{,}000})\,2^{t/10} \Rightarrow \cancel{2}^1 = \cancel{2}^{t/10} \Rightarrow$

$1 = \frac{t}{10} \Rightarrow t = 10$ ∴ After 10 years, the price will be double.

e) In what year will the price be ~~$600,000~~? $1,000,000 (one million)

(Use Desmos to answer)

∴ The value of the house will be one million at 27.37 years (2010 + 27.37) (in the year of 2037) 2037

Double check sub $x = 27.37$

$P(27.37) = (\$150{,}000)\,2^{\frac{27.37}{10}} \approx \$1{,}000{,}023.85$

Example 2. The initial number of bacteria in a sample is 400 and the number is double every 6 hours.

a) Develop a formula to model this case.

$$N = (400)\left(2^{\frac{t}{6}}\right)$$

t = time (in hours)

b) What is the number of bacteria in the sample after 2 days?

2 days = ? hours –

c) After how hours days will the number of bacteria be 6400?

d) After how many days will the number of bacteria be one million? (use Desmos to answer this part)

Example 3. Develop a formula for each case.

a) The price of a famous one million dollars picture is triple every 50 years

b) The value of a classic car sold in 1960 for $10,000 doubles every 20 years.

c) The number of cases of people infected with a virus doubles every month starting January 1st, 2020 with 10 people.

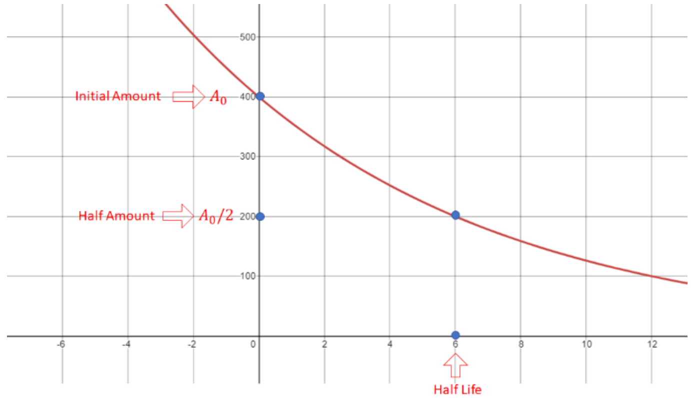
B Exponential Decay and Half Life

Exponential decay may be modelled by a formula

$$A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{H}}$$

where

- ✓ $\frac{1}{2}$ is the base
- ✓ H is called half life
- ✓ t is time
- ✓ A_0 is called the initial amount
- ✓ A is the amount at time t



Example 4. The half-life of carbon-14 is 5730 years. The relation $C = 100\% \left(\frac{1}{2}\right)^{\frac{n}{5730}}$ is used to calculate the concentration, C, in percentage, remaining n years after death.

a) Determine the carbon-14 concentration in an 10,000-year-old animal bone.

b) Determine the carbon-14 concentration in an a 50,000-year-old fossil.

c) How old is a map made from plant fibres if its carbon-14 concentration is 1%?

Example 5. A car sold in 2010 at $\$20,000$ has its value depreciated to a half every 5 years.

a) Develop a formula to model the value of the car after n years from the purchase.

b) Find the value of the car after12 years from the purchase.

c) After how many years will the value of the car be only $1,000? (you may use Desmos for this part)

Notes: Textbook Pages 406-410
Homework: Textbook Pages 410-413 # 1, 4, 6