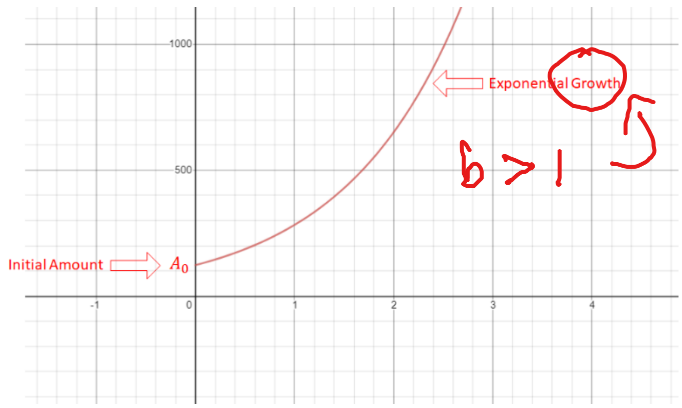
7.5 Modelling Exponential Growth and Decay

A Exponential Growth

Many situations in science and real life may be modelled by exponential relations.

Examples of exponential growths:

- ✓ saving accounts
- ✓ value of a house
- ✓ population of a city or a bacteria sample



All these situations may be modelled by a formula

$A = A_0 b^t$

where

- ✓ $b > 1$ is called the growth factor
- ✓ A_0 is called the initial amount
- ✓ t is time
- ✓ A is the amount at time t

Activity 1 (World Population Growth)

In the table below is given the world population by year from 2010 to 2020.

a) Enter this information in Desmos as a table of values (x and y numbers only)
b) Use exponential regression to find a formula for this exponential growth
c) Use this formula to estimate the world population in the year of 2100

Note. In case you get in trouble, use this link: Desmos

Year	# years after 2010 x	Population (in billions) y
2010	0	6.956
2011	1	7.041
2012	2	7.125
2013	3	7.210
2014	4	7.295
2015	5	7.379
2016	6	7.464
2017	7	7.547
2018	8	7.631
2019	9	7.713
2020	10	7.794

$a = 6.96516$
$b = 1.01144$
$y = a\,b^x$
$y = (6.96516)(1.01144)^x$
c) Year = 2100
$x = 2100 - 2010 = 90$

$y = 6.96516 \cdot 1.01144^{90} \simeq 19.388$

∴ in 2100, the world population will be around 19.4 billions of people

B Exponential Decay

Examples of exponential decays:

- ✓ Value of a car
- ✓ Radioactivity of a sample
- ✓ Intensity of light in a lake

All these situations may be modelled by a formula

$$A = A_0 b^t$$

where

- ✓ $b < 1$ is called the decay factor
- ✓ A_0 is called the initial amount
- ✓ t is time
- ✓ A is the amount at time t

Activity 2 (Car Value Decay)

In the table below is given the value of a car after purchase.

a) Enter this information in Desmos as a table of values (x and y numbers only)
b) Use exponential regression to find a formula for this exponential decay
c) Use this formula to estimate the value of the car after 10 years

Note. In case you get in trouble, use this link: Desmos

$a = 27.4155$
$b = 0.858026 < 1$
$y = a \cdot b^x$
$y = (27.4155)(0.858026)^x$

# years after purchase x	Population (in thousands of \$) y
0	27
1	24
2	20.4
3	17.34
4	14.74
5	12.53

(c) $x = 10$
$y = (27.4155)(0.858026)^{10} \simeq 5.92926$

∴ The value after 10 years is about \$5,929.26

Activity 3. Comparing Exponential Growths

Example. The projected populations, P (in thousands), of Metropolis and of Gotham City can be modelled by $P_M = 117(1.018)^n$ and $P_G = 109(1.028)^n$, where n is number of years after 2006. Use technology (graphing calculator or Desmos) to determine when the populations will be the same.

Note. In case you get in trouble, use this link: Desmos

Notes: Textbook Pages 395-401
Homework: Textbook Pages 401-405 # 1, 2, 6

