7.3 Investigate Exponential Relationships

A Compare relations by their Formula

- ✓ A linear relation has a formula of the form: $y = ax + b$
- ✓ A quadratic relation has a formula of the form:
  - $y = ax^2 + bx + c$ or
  - $y = a(x - h)^2 + k$ or
  - $y = a(x - r)(x - s)$
- ✓ An exponential relation has a formula of the form: $y = b^x$

base

exponent

Note that the variable x is part of the exponent for an exponential relation.

Example 1. Classify each of the following relations as linear, quadratic or exponential.

a) $y = x^2 - x + 1$ b) $y = -2x + 3$ c) $y = 2^x$ d) $y = (x - 1)(x + 2)$ e) $y = 10^x$

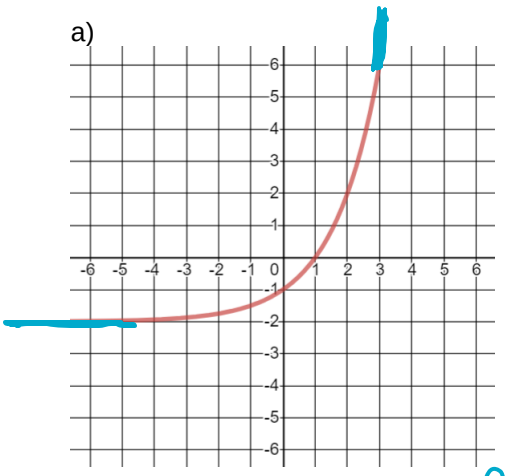
quadratic linear exponential quadratic exponential

B Compare relations by their Graphs

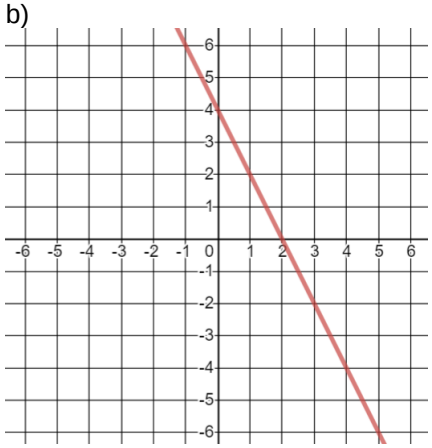
- ✓ The graph of a linear relation is a line.
- ✓ The graph of a quadratic relation is a parabola.
- ✓ The graph of an exponential relation is very steep at one end and almost horizontal at the other end.

Example 2. Classify each of the following relations as linear, quadratic or exponential.

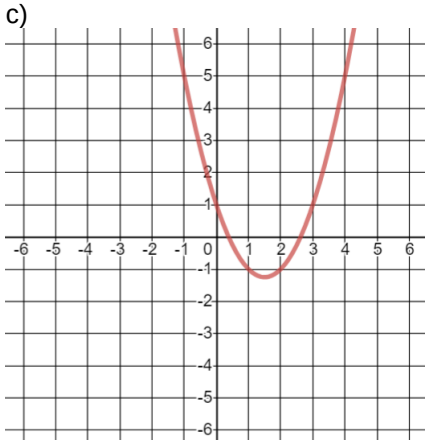
a)



b)



c)



exponential linear quadratic

C Compare relations by their Tables of Values

- ✓ The first order differences are constant for a linear relation.
- ✓ The second order differences are constant for a quadratic relation.
- ✓ The ratios of the first order differences are constant for an exponential relation.

Example 3. Classify each of the following relations as linear, quadratic or exponential.

a)

x	y	First	Second
-2	5	-3	2
-1	2	-1	2
0	1	1	2
1	2	3	
2	5		

$2-5=-3$, $1-2=-1$, $2-1=1$, $5-2=3$, $-1-(-3)=2$, $1-(-1)=2$, $3-1=2$

Constant

∴ a quadratic relation

b)

x	y	First
-2	-5	1
-1	-4	1
0	-3	1
1	-2	1
2	-1	

$-4-(-5)=1$, $-3-(-4)=1$, $-2-(-3)=1$

constant

∴ linear relation

c)

x	y	First	Ratio
-2	0.25	0.25	2
-1	0.5	0.5	2
0	1	1	2
1	2	2	
2	4		

$0.5-0.25=0.25$, $\frac{0.5}{0.25}=2$, $\frac{1}{0.5}=2$

common ratios

∴ exponential relation

D Compare relations from Real Life

Example 4. The value of a car is now (Jan 2021) $10,000. Its value depreciates by 5% each year. Make a table of value showing the values of the car for the next 5 year. Classify this relation as linear, quadratic, or exponential.

Year	Value	First Differences	Ratios
Jan 2021	$10,000		
Jan 2022	$10{,}000-\frac{5}{100}\cdot 10{,}000=9{,}500$		$\frac{9500}{10000}$
Jan 2023	$9{,}500-\frac{5}{100}\cdot 9{,}500=9{,}025$		$\frac{9025}{9500}$
Jan 2024	$9{,}025-\frac{5}{100}\cdot 9{,}025=8573.75$		same
Jan 2025	$=8145.06$		same

(skip!)

Conclusion: ∴ exponential

house

Example 5. The value of a ~~house~~ is now (Jan 2021) $100,000. Its value appreciates by $5,000 each year. Make a table of value showing the values of the house for the next 5 year. Classify this relation as linear, quadratic, or exponential.

Year	Value	First Differences	Ratios
Jan 2021	$100,000	5000	
Jan 2022	$100{,}000+5{,}000=105{,}000$	5,000	
Jan 2023	$105{,}000+5{,}000=110{,}000$	5,000	
Jan 2024	$115,000	5,000	
Jan 2025	$120,000		

constant

Conclusion: linear relation

Example 6. The population of a bacteria is today (day 1) at 1,000 and its population doubles every three days. Make a table of value showing the values of the population for the next two weeks. Classify this relation as linear, quadratic, or exponential.

Day	Population	First Differences	Ratios
Day 1	1,000		
Day 4	2,000		2
Day 7	4,000		2
Day 10	8,000		2
Day 13	16,000		2
Day 15	32,000		

Conclusion: exponential relation

Common ratios

Example 7. Julia was born in Jan 1, 2010. Her parents considered education very important for her daughter and decided to open a bank account specially designed for her future education. On the first Julia's anniversary, they deposited $1 \times \$100$ into this account. On the second anniversary, they deposited $2 \times \$100$. On the third anniversary, they deposited $3 \times \$100$, and so on. Make a table of value showing the total amount of money accumulated for the first 5 anniversaries. Classify this relation as linear, quadratic, or exponential.

Anniversary	Total amount of money	First Differences	Second Differences	Ratio
1				
2				
3				
4				
5				

Conclusion:

Example 8. A ball bouncing on the floor reaches ¾ of its previous maximum height. The initial height is 64 m. Make a table of value showing the values of the height for the next 5 bounces. Classify this relation as linear, quadratic, or exponential.

# of bounces	Height	First Differences	Ratios
0	64 m		
1	$\frac{3}{4}(64) = 48$		
2	$\frac{3}{4}(48) = 36$		
3	$\frac{3}{4}(36) = 27$		
4	$\frac{3}{4}(27) = 20.$		
5			

Conclusion:

Notes: Textbook Pages 372-376
Homework: Textbook Pages 377 # 1, 2, 3, 7, 9