

7.2 Zero and Negative Exponents

A Review of Exponent Rules

Example 1. Write as a single power (simplify). Do not evaluate.

$$\begin{aligned} \text{a) } 10^3 \times 10^7 &= 10^{3+7} = 10^{10} \\ \text{b) } a^2 \times a^1 \times a^3 &= a^{2+1+3} = a^6 \\ \text{c) } 5^{12} \div 5^7 &= \frac{5^{12}}{5^7} = 5^{12-7} = 5^5 \\ \text{d) } x^4 \div x^3 &= x^{4-3} = x^1 = x \\ \text{e) } (4^3)^5 &= 4^{3 \cdot 5} = 4^{15} \\ \text{f) } (a^6)^2 &= a^{12} \\ \text{g) } a^2 \times (a^3)^2 &= a^2 \cdot a^{3 \cdot 2} = a^2 \cdot a^6 = a^{2+6} = a^8 \\ \text{h) } (x^2)^4 \div (x^3)^2 &= \frac{x^{2 \cdot 4}}{x^{3 \cdot 2}} = \frac{x^8}{x^6} = x^{8-6} = x^2 \end{aligned}$$

B Zero Exponent

Example 2. Use two different method to simplify $a^4 \div a^4$.

$$\begin{aligned} \frac{a^4}{a^4} &= a^{4-4} = a^0 \\ \frac{a^4}{a^4} &= 1 \end{aligned}$$

$a^0 = 1$ $a \neq 0$

if $a=0$
 $0^0 = ?$
 (is undefined)

Conclusion:

$$a^0 = 1$$

if $a \neq 0$

Example 3. Write as a single power (simplify). Do not evaluate.

$$\begin{aligned} \text{a) } 10^0 \times 10^3 &= 10^{0+3} = 10^3 \\ \text{b) } a^2 \times a^0 \times a^1 &= a^{2+0+1} = a^3 \\ \text{c) } 5^2 \div 5^0 &= \frac{5^2}{5^0} = 5^{2-0} = 5^2 \\ \text{d) } (4^0)^5 &= (1)^5 = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 = 1 \\ \text{e) } (a^6)^0 &= a^{6 \cdot 0} = a^0 = 1 \\ \text{f) } (2 + 7 \times 3^5 - 4^2)^0 &= 1 \end{aligned}$$

$0^4 = 0 \cdot 0 \cdot 0 \cdot 0 = 0$

Example 4. Evaluate.

$$\begin{aligned} \text{a) } 2^0 + 3^0 - 0^4 &= 1 + 1 - 0 = 2 \\ \text{b) } (1 + 4^0 - 0^4)^2 \div (1 + 3^0) &= (1 + 1 - 0)^2 \div (1 + 1) \\ &= \frac{2^2}{2} = \frac{4}{2} = 2 \end{aligned}$$

C Negative Exponent

Example 5. Use two different method to simplify $a^2 \div a^5$.

$$\frac{a^2}{a^5} = \frac{\cancel{a} \cdot \cancel{a}}{\cancel{a} \cdot \cancel{a} \cdot a \cdot a \cdot a} = \frac{1}{a^3}$$

$$\frac{a^2}{a^5} = a^{2-5} = a^{-3}$$

Conclusion:

$$a^{-n} = \frac{1}{a^n}$$

if $a \neq 0$

Example 6. Write by using a positive exponent.

a) $10^{-2} = \frac{1}{10^2}$

b) $a^{-3} = \frac{1}{a^3}$

c) $5^{-1} = \frac{1}{5^1} = \frac{1}{5}$

Example 7. Use the exponent rules to simplify. Write the answer by using a positive exponent.

a) $10^0 \times 10^{-4} = 10^{0-4} = 10^{-4} = \frac{1}{10^4}$

b) $a^2 \times a^{-5} \times a^0 = a^{2-5+0} = a^{-3} = \frac{1}{a^3}$

c) $5^2 \div 5^5 = \frac{5^2}{5^5} = 5^{2-5} = 5^{-3} = \frac{1}{5^3}$

d) $(4^{-2})^3 = 4^{(-2) \cdot 3} = 4^{-6} = \frac{1}{4^6}$

e) $(a^{-3})^{-2} = a^{(-3) \cdot (-2)} = a^6$

Example 8. Evaluate.

find a number \rightarrow decimal
 \rightarrow integer
 \rightarrow fraction
 \downarrow
 not a power

a) $2^{-2} = \frac{1}{2^2} = \frac{1}{4} = 0.25$

b) $1^{-3} = \frac{1}{1^3} = \frac{1}{1 \cdot 1 \cdot 1} = \frac{1}{1} = 1$

c) $0^{-1} = \frac{1}{0^1} = \frac{1}{0}$ Error! (undefined)

d) $(-2)^{-3} = \frac{1}{(-2)^3} = \frac{1}{(-2) \cdot (-2) \cdot (-2)} = \frac{1}{-8} = -\frac{1}{8} = -0.125$

e) $(\frac{1}{2})^{-2} = \frac{1}{(\frac{1}{2})^2} = \frac{1}{\frac{1}{4}} = 4$
 $(\frac{2}{1})^2 = 2^2 = 4$
 $\frac{1}{0} = ?$ (undefined)

Example 9. Write as a power of 10.

a) one thousand $1000 = 10^3$

b) one thousandth $= \frac{1}{\text{thousand}} = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$

c) one billion $= 1,000,000,000 = 10^9$

d) one billionth $= \frac{1}{\text{billion}} = \frac{1}{10^9} = 10^{-9}$

Example 10. Convert into metres by using powers of 10.

a) one centimetre

b) one millimetre

c) one micron

Example 11. Write as a single power (simplify). (Challenge)

$$\text{a) } 5^{-3} \times \frac{5^6}{5^{-2}} = 5^{-3} \times 5^{6-(-2)} = 5^{-3} \times 5^8 \\ = 5^{-3+8} = 5^5$$

$$\text{b) } 3^{-5} \times (3^{-2})^{-4} = 3^{-5} \times 3^{(-2) \cdot (-4)} = 3^{-5} \times 3^8 \\ = 3^{-5+8} = 3^3$$

$$\text{c) } (7^{-4})^2 \div (7^3)^{-3} = \frac{7^{-8}}{7^{-9}} = 7^{-8-(-9)} = 7$$

$$\text{d) } \left(\left(\frac{1}{2^{-3}} \right)^{-2} \right)^{-1} = \left((2^{-3})^{-2} \right)^{-1} = 2 \\ \frac{1}{2^{-3}} = 2^3 \quad | \quad = 2^{-6} = \frac{1}{2^6}$$

Notes: Textbook Pages 364-367

Homework: Textbook Pages 368 # 2ab, 3ab, 8, 16ab, 17ab