

7.1 Exponent Rules

A Multiplication notation

Multiplication notation is shortcut for repetitive addition.

Examples:

a) $\overbrace{10 + 10 + 10} = 3(10)$

b) $x + x + x + x = 4x$

c) $\odot + \odot + \odot = 3\odot$

Example 1. Use the multiplication notation to simplify.

a) $5 + 5 + 5 + 5 + 5 + 5 = 6(5) = 30$

b) $y + y + y + y + y + y + y + y = 8y$

c) $\odot + \odot + \odot + \odot + \odot + \odot + \odot + \odot + \odot = 9 \text{ ☺}$

B Exponential notation is a shortcut for repetitive multiplication.

Examples:

a) $10 \times 10 \times 10 \times 10 \times 10 = 10^5$

b) $x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x = x^7$

c) $\blacksquare \times \blacksquare \times \blacksquare \times \blacksquare = \blacksquare^4$

Example 2. Use the exponential notation to simplify.

a) $4 \times 4 \times 4 \times 4 = 4^4$

b) $a \cdot a \cdot a \cdot a \cdot a \cdot a = a^6$

c) $\Delta \times \Delta \times \Delta \times \Delta \times \Delta \times \Delta \times \Delta = \Delta^7$

$\times \rightarrow$ times
 $\cdot \rightarrow$

$\times \times \times = \times \cdot \times \cdot \times$
 $= \times^3$
 $= \times^3$

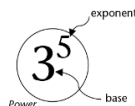
C Understanding the exponential notation

a^n

$a \Rightarrow$ base

$n \Rightarrow$ exponent

$a^n \Rightarrow$ power



D Multiplying powers with the same base

Example 3. Write as a single power and develop a rule.

$$a^4 a^3 = \underbrace{a \cdot a \cdot a \cdot a}_{a^4} \cdot \underbrace{a \cdot a \cdot a}_{a^3} = a^7 \Rightarrow a^4 \cdot a^3 = a^{4+3} = a^7$$

Conclusion:

$$a^m \times a^n = a^{m+n} \quad (1)$$

To multiply powers with the same base, keep the base the same and add the exponents.

Example 4. Write as a single power (simplify). Do not evaluate.

- a) $10^5 \times 10^3 = 10^{5+3} = 10^8$
- b) $7^4 \times 7^5 = 7^{4+5} = 7^9$
- c) $3^{10} \times 3^{21} = 3^{10+21} = 3^{31}$
- d) $x^2 \times x^6 = x^{2+6} = x^8$
- e) $2^2 \times 2^3 \times 2^4 = 2^{2+3+4} = 2^9$
- f) $a^3 \times a^4 \times a^1 \times a^5 = a^{3+4+1+5} = a^{13}$

Note. $a^1 = a$

E Dividing powers with the same base

Example 5. Write as a single power and develop a rule.

$$a^4 \div a^3 = \frac{a^4}{a^3} = \frac{a \cdot a \cdot a \cdot a}{a \cdot a \cdot a} = a = a^1 = a^{4-3}$$

$$a \div b = \frac{a}{b}$$

$$\frac{a}{a} = 1$$

Note. $a \div b = \frac{a}{b}$

Conclusion:

$$a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$$

To divide powers with the same base, keep the base the same and subtract the exponents.

Example 6. Write as a single power (simplify). Do not evaluate.

- a) $10^5 \div 10^3 = 10^{5-3} = 10^2$
- b) $7^4 \div 7^5 = 7^{4-5} = 7^{-1} = \frac{1}{7}$
- c) $3^{10} \div 3^2 = 3^{10-2} = 3^8$
- d) $x^{10} \div x^6 = x^{10-6} = x^4$

F Power of a Power

Example 5. Write as a single power and develop a rule. $6^{2 \cdot 3}$

$$(a^2)^3 = \underbrace{a^2 \cdot a^2 \cdot a^2}_{3 \text{ times}} = (a \cdot a)(a \cdot a)(a \cdot a) = a^6 = a^{2 \cdot 3}$$

Conclusion:

$$(a^m)^n = a^{m \cdot n}$$

To simplify a power of a power, keep the base the same and multiply the exponents.

Example 6. Write as a single power (simplify). Do not evaluate.

$$a) (10^3)^4 = 10^{3 \cdot 4} = 10^{12}$$

$$b) (7^2)^6 = 7^{2 \cdot 6} = 7^{12}$$

$$c) (a^5)^2 = a^{5 \cdot 2} = a^{10}$$

$$d) (x^3)^2 = x^{3 \cdot 2} = x^6$$

$$e) (x^2)^3 = x^{2 \cdot 3} = x^6$$

$$(x^3)^2 = (x^2)^3$$

$$3 \cdot 2 = 2 \cdot 3$$

Example 7. Write as a single power (simplify). (Challenge)

$$a) 5^3 \times \frac{5^6}{5^2} = 5^3 \times 5^{6-2} = 5^3 \times 5^4 = 5^{3+4} = 5^7$$

$$b) 3^5 \times (3^2)^4 = 3^5 \times 3^{2 \cdot 4} = 3^5 \times 3^8 = 3^{5+8} = 3^{13}$$

$$c) (7^4)^4 \div (7^3)^3 = 7^{4 \cdot 4} \div 7^{3 \cdot 3} = 7^{16} \div 7^9 = 7^{16-9} = 7^7$$

Notes: Textbook Pages 356-360

Homework: Textbook Pages 360 #1abc, 2abc, 3ab, 5abcd