1. Consider the line given by \( \vec{r} = (1,2,3) + t(0,1,-2) \).

   [1] a) Identify the direction vector of this line.
   \[ \vec{u} = (0,1,-2) \]

   [1] b) Find two points on this line.
   \[ t = 0 \Rightarrow A (1,2,3) \]
   \[ t = 1 \Rightarrow B (1,3,1) \]
   \[ t = 2 \Rightarrow C (3,5,-1) \]

   [1] b) Verify if the point \( P(2,3,4) \) belongs or not to the given line.
   \[ \begin{align*}
   x &= 1 + t \\
   y &= 2 + 2t \\
   z &= 3 - 2t
   \end{align*} \]
   \[ y \Rightarrow \text{The point } P(2,3,4) \text{ does not belong to the given line.} \]

2. Find the equation of a 2D line which
   [1] a) passes through the points \( A(2,-3) \) and \( B(3,-4) \)
   \[ \vec{u} = (2,-3) + t(1,1) \]

   [1] b) passes through the point \( A(0,-2) \) and is perpendicular to the vector \( \vec{v} = (-2,-5) \)
   \[ \vec{u} = (0,-2) + t(-5,-2) \]

   [1] c) passes through the point \( A(2,-4) \) and is parallel to the line \( y = -2x - 5 \)
   \[ m = -2 \Rightarrow \frac{y_2 - y_1}{x_2 - x_1} = \frac{y - y_2}{x - x_2} = \frac{1}{1} \]
   \[ x = 2 - 4 + t(1,2) \]

3. Convert the equation of the line \( \frac{x+1}{-2} = \frac{y-1}{3} = \frac{z-2}{1} \) to the vector and symmetric forms.

   [Ex/Unit Directions]
   \[ \begin{align*}
   \frac{x + 1}{-2} &= \frac{y - 1}{3} = \frac{z - 2}{1} \\
   xy - \text{int} &= 2 + t = 0 \Rightarrow t = -2 \Rightarrow \begin{cases} x = -1 - 2(-2) = 3 \\
   y = 1 + 3(-2) = -5 \end{cases} \\
   \therefore \ xy - \text{int} \text{ is } (3, -5, 0) \]
4. Find the distance from the given point to the given line.
\[ r = (-2,3) + t(3,-4), \quad B(3,-1) \]
\[
\vec{u} = (3,-4) \\
\vec{\nu} = (3,2) \\
A\vec{b} = (5,-4)
\]
\[
d = \frac{\left| A\vec{b} \cdot \vec{\nu} \right|}{\left| \vec{\nu} \right|} \\
= \frac{20-12}{\sqrt{16+9}} = \frac{8}{5}
\]
\[
\therefore d = \frac{8}{5} = 1.6
\]

5. Find the point of intersection between the two given lines.
\[ \vec{r} = (2,1,3) + t(1,2,3), \quad \vec{r} = (-3,5,-2) + s(2,-3,1) \]
\[
\begin{bmatrix}
2 + 2t &= -3 + 2s \\
1 + 2t &= 5 - 3s \\
3 + 3t &= -2 + 5s \\
9 + 9t &= -6 + 3s
\end{bmatrix}
\]
\[
10 + 11t = -1 \implies t = -1 \implies \\
\begin{cases}
x = 2 + (-1) = 1 \\
y = 1 + 2(-1) = -1 \\
z = 5 + 3(-1) = 2
\end{cases}
\]
\[ \therefore P \left( 1, -1, 2 \right) \]

6. Find the equation of the perpendicular line to the given line through the given point.
\[ \frac{x-2}{1} = \frac{y+3}{-2}, \quad B(2,3) \]
\[
2x + 4 = 5t \Rightarrow \\
2x - 5t + 4 = 0 \\
\vec{\nu} = (2,5,-1)
\]
\[
\therefore \vec{u} = (2,1,3) + t(-2,5,-1)
\]
7. Find if the lines are parallel or not. In the case the lines are parallel, find if they are coincident or not.

In the case the lines are parallel and distinct, find the distance between the lines. [A 4 marks]

\( \vec{r} = (1,2,3) + t(2,-1,0), \quad \vec{s} = (3,2,1) + s(-6,3,0) \)

\[
\vec{u} = (2, -1, 0) \quad \vec{v} = (-6, 3, 0) \Rightarrow \vec{w} = -3\vec{u}
\]

\[
\vec{AB} = (3, 2, 1) - (1, 2, 3) = (2, 0, -2)
\]

\[
\vec{AB} \times \vec{u} = \begin{vmatrix}
  \vec{i} & \vec{j} & \vec{k} \\
  2 & 0 & -2 \\
  2 & -1 & 0
\end{vmatrix} = \begin{vmatrix}
  \vec{i} & \vec{j} & \vec{k} \\
  0 & -2 & -2 \\
  -1 & 0 & -2
\end{vmatrix}
\]

\[
\begin{align*}
\vec{AB} \times \vec{u} &= (3, -4, -2) \\
\vec{w} &= (-2, 4, 2)
\end{align*}
\]

\[
\text{d} = \frac{|\vec{AB} \times \vec{u}|}{\vec{u}} = \frac{\sqrt{4+16+4}}{\sqrt{4+0+4}} = \frac{\sqrt{24}}{\sqrt{8}} = 2.19
\]

\[
\therefore \text{d} = \frac{\sqrt{24}}{5} \approx 2.19
\]

8. Find the equation of the perpendicular line from the origin \(O(0,0,0)\) to the line \( \vec{r} = (1,2,3) + t(-2,1,-3) \). [T/I 3 marks]

\[
\vec{OP} = (1 - 2t, 2 + t, 3 - 3t)
\]

\[
\vec{u} = (-2, 1, -3)
\]

\[
\vec{OP} \cdot \vec{u} = 0
\]

\[
-2s + 4t + 2 + t - 9 + 3t = 0
\]

\[
14t = 9
\]

\[
t = \frac{9}{14}
\]

\[
\vec{OP} = (1 - \frac{18}{14}, 2 + \frac{9}{14}, 3 - \frac{27}{14})
\]

\[
= (-\frac{4}{14}, \frac{37}{14}, \frac{15}{14})
\]

\[
\vec{r} = \hat{t} \vec{OP}
\]

\[
\vec{r} = \frac{9}{14} (-4, 37, 15)
\]