1. Find the point of tangency and the equation of the tangent line passing through the point $P(6, e^6)$ to the graph of the function: $f(x) = e^x$.

2. Find the point of tangency and the equation of the tangent line passing through the point $P(6, \sin(6))$ to the graph of the function: $f(x) = \sin(x)$.

3. Find the point of tangency and the equation of the tangent line passing through the point $P(2, 5^2)$ to the graph of the function: $f(x) = 5^x$.

4. Find the point of tangency and the equation of the tangent line passing through the point $P(-2, e^{-2})$ to the graph of the function: $f(x) = e^x$.

5. Find the point of tangency and the equation of the tangent line passing through the point $P(-2, \sin(-2))$ to the graph of the function: $f(x) = \sin(x)$.

6. Find the point of tangency and the equation of the tangent line passing through the point $P(3, \log 3)$ to the graph of the function: $f(x) = \log x$.

7. Find the point of tangency and the equation of the tangent line passing through the point $P(4, 5^4)$ to the graph of the function: $f(x) = 5^x$.

8. Find the point of tangency and the equation of the tangent line passing through the point $P(-5, \sin(-5))$ to the graph of the function: $f(x) = \sin(x)$.

9. Find the point of tangency and the equation of the tangent line passing through the point $P(-5, 10^{-5})$ to the graph of the function: $f(x) = 10^x$.

10. Find the point of tangency and the equation of the tangent line passing through the point $P(5, 4^5)$ to the graph of the function: $f(x) = 4^x$.

Answers:

1. $y = 403.429x - 2,017.144$
2. $y = 0.960x - 6.040$
3. $y = 40.236x - 55.472$
4. $y = 0.135x + 0.406$
5. $y = -0.416x - 1.742$
6. $y = 0.145x + 0.043$
7. $y = 1,005.899x - 3,398.595$
8. $y = 0.284x + 2.377$
9. $y = 0.000x + 0.000$
10. $y = 1,419.565x - 6,073.827$

Solutions:

1. $P(6, e^6)$ $f(x) = e^x$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$

$x_1 = 6$ $y_1 = e^6 = 403.429$ $m_1 = f'(x_1) = 403.429$

$y - 403.429 = 403.429(x - 6)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 403.429x - 2,017.144$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

2. $P(6, \sin(6))$ $f(x) = \sin(x)$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos x$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$

$x_1 = 6$ $y_1 = \sin(6) = -0.279$ $m_1 = f'(x_1) = 0.960$

$y - -0.279 = 0.960(x - 6)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 0.960x - 6.040$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

3. $P(2, 5^2)$ $f(x) = 5^x$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} 5^x = (\ln 5)(5^x)$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} b^x = (\ln b) b^x$

$x_1 = 2$ $y_1 = 5^2 = 25.000$ $m_1 = f'(x_1) = 40.236$

$y - 25.000 = 40.236(x - 2)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 40.236x - 55.472$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

4. $P(-2, e^{-2})$ $f(x) = e^x$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} e^x = e^x$

$x_1 = -2$ $y_1 = e^{-2} = 0.135$ $m_1 = f'(x_1) = 0.135$

$y - 0.135 = 0.135(x - -2)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 0.135x + 0.406$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

5. $P(-2, \sin(-2))$ $f(x) = \sin(x)$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos x$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$

$x_1 = -2$ $y_1 = \sin(-2) = -0.909$ $m_1 = f'(x_1) = -0.416$

$y - -0.909 = -0.416(x - -2)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = -0.416x - 1.742$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

6. $P(3, \log 3)$ $f(x) = \log x$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} \log x = \dfrac{1}{(\ln 10)x}$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} \log x = \dfrac{1}{(\ln 10)x}$

$x_1 = 3$ $y_1 = \log 3 = 0.477$ $m_1 = f'(x_1) = 0.145$

$y - 0.477 = 0.145(x - 3)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 0.145x + 0.043$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

7. $P(4, 5^4)$ $f(x) = 5^x$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} 5^x = (\ln 5)(5^x)$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} b^x = (\ln b) b^x$

$x_1 = 4 \qquad y_1 = 5^4 = 625.000 \qquad m_1 = f'(x_1) = 1,005.899$

$y - 625.000 = 1,005.899(x - 4)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 1,005.899x - 3,398.595$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

8. $P(-5, \sin(-5)) \qquad f(x) = \sin(x)$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} \sin(x) = \cos x$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$

$x_1 = -5 \qquad y_1 = \sin(-5) = 0.959 \qquad m_1 = f'(x_1) = 0.284$

$y - 0.959 = 0.284(x - -5)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 0.284x + 2.377$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

9. $P(-5, 10^{-5}) \qquad f(x) = 10^x$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} 10^x = (\ln 10)(10^x)$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} 10^x = (\ln 10)10^x$

$x_1 = -5 \qquad y_1 = 10^{-5} = 0.000 \qquad m_1 = f'(x_1) = 0.000$

$y - 0.000 = 0.000(x - -5)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 0.000x + 0.000$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

10. $P(5, 4^5) \qquad f(x) = 4^x$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x} 4^x = (\ln 4)(4^x)$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x} b^x = (\ln b)b^x$

$x_1 = 5 \qquad y_1 = 4^5 = 1,024.000 \qquad m_1 = f'(x_1) = 1,419.565$

$y - 1,024.000 = 1,419.565(x - 5)$ ◀ Apply: $y - y_1 = m(x - x_1)$

$\therefore y = 1,419.565x - 6,073.827$ ◀ Write the equation of the tangent line in the form: $y = mx + b$

