1. Differentiate: $f(x) = (\sin x)^{\sqrt{x}}$

2. Differentiate: $f(x) = (\cos x)^{\cos x}$

3. Differentiate: $f(x) = \left(\sqrt{x}\right)^{x}$

4. Differentiate: $f(x) = (\ln x)^{\log x}$

5. Differentiate: $f(x) = (\cos x)^{\sqrt{x}}$

6. Differentiate: $f(x) = \left(\sqrt{x}\right)^{\ln x}$

7. Differentiate: $f(x) = \left(\sqrt{x}\right)^{\sin x}$

8. Differentiate: $f(x) = (\log x)^{\sin x}$

9. Differentiate: $f(x) = (\cos x)^{\sin x}$

10. Differentiate: $f(x) = (\ln x)^{x}$

Answers:

1. $f'(x) = (\sin x)^{\sqrt{x}} \left[\frac{1}{2\sqrt{x}} \ln \sin x + \cos x \frac{\sqrt{x}}{\sin x} \right]$

2. $f'(x) = (\cos x)^{\cos x} [\ln \cos x + 1] (-\sin x)$

3. $f'(x) = \left(\sqrt{x}\right)^{x} \left[\ln \sqrt{x} + \frac{1}{2} \right]$

4. $f'(x) = (\ln x)^{\log x} \left[\frac{1}{(\ln 10)x} \ln \ln x + \frac{1}{x} \frac{\log x}{\ln x} \right]$

5. $f'(x) = (\cos x)^{\sqrt{x}} \left[\frac{1}{2\sqrt{x}} \ln \cos x + (-\sin x) \frac{\sqrt{x}}{\cos x} \right]$

6. $f'(x) = \left(\sqrt{x}\right)^{\ln x} \left[\frac{1}{x} \ln \sqrt{x} + \frac{\ln x}{2x} \right]$

7. $f'(x) = \left(\sqrt{x}\right)^{\sin x} \left[\cos x \ln \sqrt{x} + \frac{\sin x}{2x} \right]$

8. $f'(x) = (\log x)^{\sin x} \left[\cos x \ln \log x + \frac{1}{(\ln 10)x} \frac{\sin x}{\log x} \right]$

9. $f'(x) = (\cos x)^{\sin x} \left[\cos x \ln \cos x + (-\sin x) \frac{\sin x}{\cos x} \right]$

10. $f'(x) = (\ln x)^{x} \left[\ln \ln x + \frac{1}{\ln x} \right]$

Solutions:

1. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} f(x) = \frac{\mathrm{d}}{\mathrm{d}x} (\sin x)^{\sqrt{x}}$

Write the original function as a power using the identity: $f^g = e^{g \ln f}$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} e^{\sqrt{x} \ln \sin x}$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} e^{f(x)} = e^{f(x)} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$

$= e^{\sqrt{x} \ln \sin x} \frac{\mathrm{d}}{\mathrm{d}x} \sqrt{x} \ln \sin x$ ◄ Apply: $f^g = e^{g \ln f}$ $\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = g(x)\frac{\mathrm{d}}{\mathrm{d}x} f(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x} g(x)$

$= (\sin x)^{\sqrt{x}} \left[\ln \sin x \frac{\mathrm{d}}{\mathrm{d}x} \sqrt{x} + \sqrt{x} \frac{\mathrm{d}}{\mathrm{d}x} \ln \sin x\right]$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} \sqrt{x} = \frac{1}{2\sqrt{x}}$ $\frac{\mathrm{d}}{\mathrm{d}x} \ln f(x) = \frac{1}{f(x)} f'(x)$

$= (\sin x)^{\sqrt{x}} \left[\frac{1}{2\sqrt{x}} \ln \sin x + \sqrt{x} \frac{1}{\sin x} \frac{\mathrm{d}}{\mathrm{d}x} \sin x\right]$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} \sin x = \cos x$

$= (\sin x)^{\sqrt{x}} \left[\frac{1}{2\sqrt{x}} \ln \sin x + \cos x \frac{\sqrt{x}}{\sin x}\right]$ ◄ Simplify, if necessary.

$\therefore \frac{\mathrm{d}}{\mathrm{d}x} (\sin x)^{\sqrt{x}} = (\sin x)^{\sqrt{x}} \left[\frac{1}{2\sqrt{x}} \ln \sin x + \cos x \frac{\sqrt{x}}{\sin x}\right]$

2. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} f(x) = \frac{\mathrm{d}}{\mathrm{d}x} (\cos x)^{\cos x}$

Write the original function as a power using the identity: $f^g = e^{g \ln f}$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} e^{\cos x \ln \cos x}$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} e^{f(x)} = e^{f(x)} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$

$= e^{\cos x \ln \cos x} \frac{\mathrm{d}}{\mathrm{d}x} \cos x \ln \cos x$ ◄ Apply: $f^g = e^{g \ln f}$ $\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = g(x)\frac{\mathrm{d}}{\mathrm{d}x} f(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x} g(x)$

$= (\cos x)^{\cos x} \left[\ln \cos x \frac{\mathrm{d}}{\mathrm{d}x} \cos x + \cos x \frac{\mathrm{d}}{\mathrm{d}x} \ln \cos x\right]$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} \cos x = -\sin x$ $\frac{\mathrm{d}}{\mathrm{d}x} \ln f(x) =$
$\frac{1}{f(x)} f'(x)$

$= (\cos x)^{\cos x} \left[(-\sin x) \ln \cos x + \cos x \frac{1}{\cos x} \frac{\mathrm{d}}{\mathrm{d}x} \cos x\right]$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} \cos x = -\sin x$

$= (\cos x)^{\cos x} \left[(-\sin x) \ln \cos x + (-\sin x) \frac{\cos x}{\cos x}\right]$ ◄ Simplify, if necessary.

$\therefore \frac{\mathrm{d}}{\mathrm{d}x} (\cos x)^{\cos x} = (\cos x)^{\cos x} [\ln \cos x + 1] (-\sin x)$

3. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} f(x) = \frac{\mathrm{d}}{\mathrm{d}x} \left(\sqrt{x}\right)^x$

Write the original function as a power using the identity: $f^g = e^{g \ln f}$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x} e^{x \ln \sqrt{x}}$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} e^{f(x)} = e^{f(x)} \frac{\mathrm{d}}{\mathrm{d}x} f(x)$

$= e^{x \ln \sqrt{x}} \frac{\mathrm{d}}{\mathrm{d}x} x \ln \sqrt{x}$ ◄ Apply: $f^g = e^{g \ln f}$ $\frac{\mathrm{d}}{\mathrm{d}x} f(x)g(x) = g(x)\frac{\mathrm{d}}{\mathrm{d}x} f(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x} g(x)$

$= \left(\sqrt{x}\right)^x \left[\ln \sqrt{x} \frac{\mathrm{d}}{\mathrm{d}x} x + x \frac{\mathrm{d}}{\mathrm{d}x} \ln \sqrt{x}\right]$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} x = 1$ $\frac{\mathrm{d}}{\mathrm{d}x} \ln f(x) = \frac{1}{f(x)} f'(x)$

$= \left(\sqrt{x}\right)^x \left[\ln \sqrt{x} + x \frac{1}{\sqrt{x}} \frac{\mathrm{d}}{\mathrm{d}x} \sqrt{x}\right]$ ◄ Apply: $\frac{\mathrm{d}}{\mathrm{d}x} \sqrt{x} = \frac{1}{2\sqrt{x}}$

$= \left(\sqrt{x}\right)^x \left[\ln \sqrt{x} + \frac{1}{2\sqrt{x}} \frac{x}{\sqrt{x}}\right]$ ◄ Simplify, if necessary.

$\therefore \frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x}\right)^x = \left(\sqrt{x}\right)^x \left[\ln\sqrt{x} + \frac{1}{2}\right]$

4. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}f(x) = \frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^{\log x}$

Write the original function as a power using the identity: $f^g = e^{g\ln f}$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}e^{\log x \ln\ln x}$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}e^{f(x)} = e^{f(x)}\frac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= e^{\log x \ln\ln x}\frac{\mathrm{d}}{\mathrm{d}x}\log x\ln\ln x$ ◀ Apply: $f^g = e^{g\ln f}$ $\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)$

$= (\ln x)^{\log x}\left[\ln\ln x\frac{\mathrm{d}}{\mathrm{d}x}\log x + \log x\frac{\mathrm{d}}{\mathrm{d}x}\ln\ln x\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}\log x = \frac{1}{(\ln 10)x}$ $\frac{\mathrm{d}}{\mathrm{d}x}\ln f(x) = \frac{1}{f(x)}f'(x)$

$= (\ln x)^{\log x}\left[\frac{1}{(\ln 10)x}\ln\ln x + \log x\frac{1}{\ln x}\frac{\mathrm{d}}{\mathrm{d}x}\ln x\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$

$= (\ln x)^{\log x}\left[\frac{1}{(\ln 10)x}\ln\ln x + \frac{1}{x}\frac{\log x}{\ln x}\right]$ ◀ Simplify, if necessary.

$\therefore \frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^{\log x} = (\ln x)^{\log x}\left[\frac{1}{(\ln 10)x}\ln\ln x + \frac{1}{x}\frac{\log x}{\ln x}\right]$

5. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}f(x) = \frac{\mathrm{d}}{\mathrm{d}x}(\cos x)^{\sqrt{x}}$

Write the original function as a power using the identity: $f^g = e^{g\ln f}$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}e^{\sqrt{x}\ln\cos x}$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}e^{f(x)} = e^{f(x)}\frac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= e^{\sqrt{x}\ln\cos x}\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x}\ln\cos x$ ◀ Apply: $f^g = e^{g\ln f}$ $\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)$

$= (\cos x)^{\sqrt{x}}\left[\ln\cos x\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} + \sqrt{x}\frac{\mathrm{d}}{\mathrm{d}x}\ln\cos x\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} = \frac{1}{2\sqrt{x}}$ $\frac{\mathrm{d}}{\mathrm{d}x}\ln f(x) = \frac{1}{f(x)}f'(x)$

$= (\cos x)^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln\cos x + \sqrt{x}\frac{1}{\cos x}\frac{\mathrm{d}}{\mathrm{d}x}\cos x\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

$= (\cos x)^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln\cos x + (-\sin x)\frac{\sqrt{x}}{\cos x}\right]$ ◀ Simplify, if necessary.

$\therefore \frac{\mathrm{d}}{\mathrm{d}x}(\cos x)^{\sqrt{x}} = (\cos x)^{\sqrt{x}}\left[\frac{1}{2\sqrt{x}}\ln\cos x + (-\sin x)\frac{\sqrt{x}}{\cos x}\right]$

6. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}f(x) = \frac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x}\right)^{\ln x}$

Write the original function as a power using the identity: $f^g = e^{g\ln f}$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}e^{\ln x\ln\sqrt{x}}$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}e^{f(x)} = e^{f(x)}\frac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= e^{\ln x\ln\sqrt{x}}\frac{\mathrm{d}}{\mathrm{d}x}\ln x\ln\sqrt{x}$ ◀ Apply: $f^g = e^{g\ln f}$ $\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)$

$= \left(\sqrt{x}\right)^{\ln x}\left[\ln\sqrt{x}\frac{\mathrm{d}}{\mathrm{d}x}\ln x + \ln x\frac{\mathrm{d}}{\mathrm{d}x}\ln\sqrt{x}\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$ $\frac{\mathrm{d}}{\mathrm{d}x}\ln f(x) = \frac{1}{f(x)}f'(x)$

$= \left(\sqrt{x}\right)^{\ln x}\left[\frac{1}{x}\ln\sqrt{x} + \ln x\frac{1}{\sqrt{x}}\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x}\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} = \frac{1}{2\sqrt{x}}$

$= \left(\sqrt{x}\right)^{\ln x}\left[\frac{1}{x}\ln\sqrt{x} + \frac{1}{2\sqrt{x}}\frac{\ln x}{\sqrt{x}}\right]$ ◀ Simplify, if necessary.

$\therefore \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x}\right)^{\ln x} = \left(\sqrt{x}\right)^{\ln x}\left[\dfrac{1}{x}\ln\sqrt{x} + \dfrac{\ln x}{2x}\right]$

7. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x}\right)^{\sin x}$

Write the original function as a power using the identity: $f^g = e^{g\ln f}$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}e^{\sin x\ln\sqrt{x}}$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{f(x)} = e^{f(x)}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= e^{\sin x\ln\sqrt{x}}\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x\ln\sqrt{x}$ ◀ Apply: $f^g = e^{g\ln f}$ $\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\dfrac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\dfrac{\mathrm{d}}{\mathrm{d}x}g(x)$

$= \left(\sqrt{x}\right)^{\sin x}\left[\ln\sqrt{x}\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x + \sin x\dfrac{\mathrm{d}}{\mathrm{d}x}\ln\sqrt{x}\right]$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$ $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln f(x) = \dfrac{1}{f(x)}f'(x)$

$= \left(\sqrt{x}\right)^{\sin x}\left[\cos x\ln\sqrt{x} + \sin x\dfrac{1}{\sqrt{x}}\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{x}\right]$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}\sqrt{x} = \dfrac{1}{2\sqrt{x}}$

$= \left(\sqrt{x}\right)^{\sin x}\left[\cos x\ln\sqrt{x} + \dfrac{1}{2\sqrt{x}}\dfrac{\sin x}{\sqrt{x}}\right]$ ◀ Simplify, if necessary.

$\therefore \dfrac{\mathrm{d}}{\mathrm{d}x}\left(\sqrt{x}\right)^{\sin x} = \left(\sqrt{x}\right)^{\sin x}\left[\cos x\ln\sqrt{x} + \dfrac{\sin x}{2x}\right]$

8. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(\log x)^{\sin x}$

Write the original function as a power using the identity: $f^g = e^{g\ln f}$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}e^{\sin x\ln\log x}$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{f(x)} = e^{f(x)}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= e^{\sin x\ln\log x}\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x\ln\log x$ ◀ Apply: $f^g = e^{g\ln f}$ $\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\dfrac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\dfrac{\mathrm{d}}{\mathrm{d}x}g(x)$

$= (\log x)^{\sin x}\left[\ln\log x\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x + \sin x\dfrac{\mathrm{d}}{\mathrm{d}x}\ln\log x\right]$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$ $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln f(x) = \dfrac{1}{f(x)}f'(x)$

$= (\log x)^{\sin x}\left[\cos x\ln\log x + \sin x\dfrac{1}{\log x}\dfrac{\mathrm{d}}{\mathrm{d}x}\log x\right]$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}\log x = \dfrac{1}{(\ln 10)x}$

$= (\log x)^{\sin x}\left[\cos x\ln\log x + \dfrac{1}{(\ln 10)x}\dfrac{\sin x}{\log x}\right]$ ◀ Simplify, if necessary.

$\therefore \dfrac{\mathrm{d}}{\mathrm{d}x}(\log x)^{\sin x} = (\log x)^{\sin x}\left[\cos x\ln\log x + \dfrac{1}{(\ln 10)x}\dfrac{\sin x}{\log x}\right]$

9. $f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}f(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}(\cos x)^{\sin x}$

Write the original function as a power using the identity: $f^g = e^{g\ln f}$

$f'(x) = \dfrac{\mathrm{d}}{\mathrm{d}x}e^{\sin x\ln\cos x}$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}e^{f(x)} = e^{f(x)}\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= e^{\sin x\ln\cos x}\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x\ln\cos x$ ◀ Apply: $f^g = e^{g\ln f}$ $\dfrac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\dfrac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\dfrac{\mathrm{d}}{\mathrm{d}x}g(x)$

$= (\cos x)^{\sin x}\left[\ln\cos x\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x + \sin x\dfrac{\mathrm{d}}{\mathrm{d}x}\ln\cos x\right]$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}\sin x = \cos x$ $\dfrac{\mathrm{d}}{\mathrm{d}x}\ln f(x) = \dfrac{1}{f(x)}f'(x)$

$= (\cos x)^{\sin x}\left[\cos x\ln\cos x + \sin x\dfrac{1}{\cos x}\dfrac{\mathrm{d}}{\mathrm{d}x}\cos x\right]$ ◀ Apply: $\dfrac{\mathrm{d}}{\mathrm{d}x}\cos x = -\sin x$

$= (\cos x)^{\sin x}\left[\cos x\ln\cos x + (-\sin x)\dfrac{\sin x}{\cos x}\right]$ ◀ Simplify, if necessary.

$\therefore \frac{\mathrm{d}}{\mathrm{d}x}(\cos x)^{\sin x} = (\cos x)^{\sin x}\left[\cos x \ln\cos x + (-\sin x)\frac{\sin x}{\cos x}\right]$

10. $f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}f(x) = \frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^x$

Write the original function as a power using the identity: $f^g = e^{g\ln f}$

$f'(x) = \frac{\mathrm{d}}{\mathrm{d}x}e^{x\ln\ln x}$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}e^{f(x)} = e^{f(x)}\frac{\mathrm{d}}{\mathrm{d}x}f(x)$

$= e^{x\ln\ln x}\frac{\mathrm{d}}{\mathrm{d}x}x\ln\ln x$ ◀ Apply: $f^g = e^{g\ln f}$ $\frac{\mathrm{d}}{\mathrm{d}x}f(x)g(x) = g(x)\frac{\mathrm{d}}{\mathrm{d}x}f(x) + f(x)\frac{\mathrm{d}}{\mathrm{d}x}g(x)$

$= (\ln x)^x\left[\ln\ln x\frac{\mathrm{d}}{\mathrm{d}x}x + x\frac{\mathrm{d}}{\mathrm{d}x}\ln\ln x\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}x = 1$ $\frac{\mathrm{d}}{\mathrm{d}x}\ln f(x) = \frac{1}{f(x)}f'(x)$

$= (\ln x)^x\left[\ln\ln x + x\frac{1}{\ln x}\frac{\mathrm{d}}{\mathrm{d}x}\ln x\right]$ ◀ Apply: $\frac{\mathrm{d}}{\mathrm{d}x}\ln x = \frac{1}{x}$

$= (\ln x)^x\left[\ln\ln x + \frac{1}{x}\frac{x}{\ln x}\right]$ ◀ Simplify, if necessary.

$\therefore \frac{\mathrm{d}}{\mathrm{d}x}(\ln x)^x = (\ln x)^x\left[\ln\ln x + \frac{1}{\ln x}\right]$