

1. Differentiate: $f(x) = \log_2 \sin x$
2. Differentiate: $f(x) = \cos \sin x$
3. Differentiate: $f(x) = \log_4 10^x$
4. Differentiate: $f(x) = \cos 5^x$
5. Differentiate: $f(x) = \ln \cos x$
6. Differentiate: $f(x) = \sin \sin x$
7. Differentiate: $f(x) = 3^{2^x}$
8. Differentiate: $f(x) = \log \log_4 x$
9. Differentiate: $f(x) = 5^{10^x}$
10. Differentiate: $f(x) = 3^{10^x}$

ANSWERS:

1. $f'(x) = \frac{\ln 2}{\cos x} \sin x - \cos x \sin x$
2. $f'(x) = -\sin \cos x$
3. $f'(x) = \log_4 10 = \frac{1}{\ln 4} \ln 10$
4. $f'(x) = -\sin 5^x \ln 5 \cdot 5^x$
5. $f'(x) = \frac{x \cos x}{\sin x}$
6. $f'(x) = \cos \sin x$
7. $f'(x) = 2^x \ln 2 \cdot 3^{2^x}$
8. $f'(x) = \frac{x \ln 3}{\ln 4}$
9. $f'(x) = 10^{10^x} \ln 5$
10. $f'(x) = 10^{10^x} \ln 3$

Solutions:

$$1. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \log_2 \sin x \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \log_b f(x) = \frac{1}{(\ln b)f(x)} f'(x)$$

$$= \left(\frac{1}{(\ln 2) \sin x} \right) \frac{d}{dx} (\sin x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \sin x = \cos x$$

$$= \left(\frac{1}{(\ln 2) \sin x} \right) (\cos x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} \log_2 \sin x = \frac{\cos x}{(\ln 2) \sin x}$$

$$2. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \cos \sin x \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \cos f(x) = -(\sin f(x))f'(x)$$

$$= (-\sin \sin x) \frac{d}{dx} (\sin x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \sin x = \cos x$$

$$= (-\sin \sin x) (\cos x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} \cos \sin x = -\cos x \sin \sin x$$

$$3. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \log_4 10^x \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \log_b f(x) = \frac{1}{(\ln b)f(x)} f'(x)$$

$$= \left(\frac{1}{(\ln 4)10^x} \right) \frac{d}{dx} (10^x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} 10^x = (\ln 10)10^x$$

$$= \left(\frac{1}{(\ln 4)10^x} \right) ((\ln 10)10^x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} \log_4 10^x = \log_4 10$$

$$4. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \cos 5^x \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \cos f(x) = -(\sin f(x))f'(x)$$

$$= (-\sin 5^x) \frac{d}{dx} (5^x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} b^x = (\ln b)b^x$$

$$= (-\sin 5^x) ((\ln 5)5^x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} \cos 5^x = -(\ln 5)5^x \sin 5^x$$

$$5. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \ln \cos x \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \ln f(x) = \frac{1}{f(x)} f'(x)$$

$$= \left(\frac{1}{\cos x} \right) \frac{d}{dx} (\cos x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \cos x = -\sin x$$

$$= \left(\frac{1}{\cos x} \right) (-\sin x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} \ln \cos x = \frac{-\sin x}{\cos x}$$

$$6. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \sin \sin x \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \sin f(x) = (\cos f(x))f'(x)$$

$$= (\cos \sin x) \frac{d}{dx} (\sin x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \sin x = \cos x$$

$$= (\cos \sin x) (\cos x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} \sin \sin x = \cos x \cos \sin x$$

$$7. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} 3^{2^x} \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} b^{f(x)} = (\ln b) b^{f(x)} f'(x)$$

$$= \left((\ln 3) 3^{2^x} \right) \frac{d}{dx} (2^x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} b^x = (\ln b) b^x$$

$$= \left((\ln 3) 3^{2^x} \right) ((\ln 2) 2^x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} 3^{2^x} = (\ln 2) 2^x (\ln 3) 3^{2^x}$$

$$8. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \log \log_4 x \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \log f(x) = \frac{1}{(\ln 10) f(x)} f'(x)$$

$$= \left(\frac{1}{(\ln 10) \log_4 x} \right) \frac{d}{dx} (\log_4 x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} \log_b x = \frac{1}{(\ln b)x}$$

$$= \left(\frac{1}{(\ln 10) \log_4 x} \right) \left(\frac{1}{(\ln 4)x} \right) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} \log \log_4 x = \frac{\frac{1}{(\ln 4)x}}{(\ln 10) \log_4 x}$$

$$9. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} 5^{10^x} \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} b^{f(x)} = (\ln b) b^{f(x)} f'(x)$$

$$= \left((\ln 5) 5^{10^x} \right) \frac{d}{dx} (10^x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} 10^x = (\ln 10) 10^x$$

$$= \left((\ln 5) 5^{10^x} \right) ((\ln 10) 10^x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} 5^{10^x} = (\ln 10) 10^x (\ln 5) 5^{10^x}$$

$$10. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} 3^{10^x} \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} b^{f(x)} = (\ln b) b^{f(x)} f'(x)$$

$$= \left((\ln 3) 3^{10^x} \right) \frac{d}{dx} (10^x) \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} 10^x = (\ln 10) 10^x$$

$$= \left((\ln 3) 3^{10^x} \right) ((\ln 10) 10^x) \quad \blacktriangleleft \text{Simplify, if necessary.}$$

$$\therefore \frac{d}{dx} 3^{10^x} = (\ln 10) 10^x (\ln 3) 3^{10^x}$$