

1. Differentiate:  $f(x) = \sqrt[6]{\ln x}$

2. Differentiate:  $f(x) = \sqrt[7]{e^x}$

3. Differentiate:  $f(x) = \frac{1}{\sqrt[8]{e^x}}$

4. Differentiate:  $f(x) = (3^x)^7$

5. Differentiate:  $f(x) = \frac{1}{\sqrt[7]{\cos x}}$

6. Differentiate:  $f(x) = \sin^7 x$

7. Differentiate:  $f(x) = \cos^6 x$

8. Differentiate:  $f(x) = \frac{1}{\ln^3 x}$

9. Differentiate:  $f(x) = (4^x)^6$

10. Differentiate:  $f(x) = \frac{1}{(e^x)^8}$

Answers:

1.  $\frac{1}{6x\sqrt[6]{\ln^5 x}}$
2.  $\frac{7}{e^{\frac{x}{7}}}$
3.  $-\frac{8e^{\frac{x}{8}}}{8}$
4.  $7(\ln 3)3^{7x}$
5.  $\frac{7\sqrt[7]{\cos^8 x}}{\sin x}$
6.  $7 \cos x \sin^6 x$
7.  $-6 \sin x \cos^5 x$
8.  $-\frac{3}{x \ln^4 x}$
9.  $6(\ln 4)4^{6x}$
10.  $-\frac{e^{\frac{x}{8}}}{8}$

Solutions:

$$\begin{aligned}
1. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \sqrt[6]{\ln x} &< \text{Write the function as a power:} \\
&= \frac{d}{dx} (\ln x)^{\frac{1}{6}} &< \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x) \\
&= \frac{1}{6} (\ln x)^{\frac{1}{6}-1} \frac{d}{dx} \ln x &< \text{Simplify and apply: } \frac{d}{dx} \ln x = \frac{1}{x} \\
&= \frac{1}{6} (\ln x)^{-\frac{5}{6}} \frac{1}{x} &< \text{Simplify:} \\
\therefore \frac{d}{dx} \sqrt[6]{\ln x} &= \frac{1}{6x \sqrt[6]{\ln^5 x}}
\end{aligned}$$

$$\begin{aligned}
2. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \sqrt[7]{e^x} &< \text{Write the function as a power:} \\
&= \frac{d}{dx} (e^x)^{\frac{1}{7}} &< \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x) \\
&= \frac{1}{7} (e^x)^{\frac{1}{7}-1} \frac{d}{dx} e^x &< \text{Simplify and apply: } \frac{d}{dx} e^x = e^x \\
&= \frac{1}{7} (e^x)^{-\frac{6}{7}} e^x &< \text{Simplify:} \\
\therefore \frac{d}{dx} \sqrt[7]{e^x} &= \frac{e^{\frac{x}{7}}}{7}
\end{aligned}$$

$$\begin{aligned}
3. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{1}{\sqrt[8]{e^x}} &< \text{Write the function as a power:} \\
&= \frac{d}{dx} (e^x)^{-\frac{1}{8}} &< \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x) \\
&= -\frac{1}{8} (e^x)^{-\frac{1}{8}-1} \frac{d}{dx} e^x &< \text{Simplify and apply: } \frac{d}{dx} e^x = e^x \\
&= -\frac{1}{8} (e^x)^{-\frac{9}{8}} e^x &< \text{Simplify:} \\
\therefore \frac{d}{dx} \frac{1}{\sqrt[8]{e^x}} &= \frac{-1}{8e^{\frac{x}{8}}}
\end{aligned}$$

$$\begin{aligned}
4. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} (3^x)^7 &< \text{Write the function as a power:} \\
&= \frac{d}{dx} (3^x)^7 &< \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x) \\
&= 7(3^x)^{7-1} \frac{d}{dx} (3)^x &< \text{Simplify and apply: } \frac{d}{dx} b^x = (\ln b)b^x \\
&= 7(3^x)^6 (\ln 3)(3)^x &< \text{Simplify:} \\
\therefore \frac{d}{dx} (3^x)^7 &= 7(\ln 3)3^{7x}
\end{aligned}$$

$$\begin{aligned}
5. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{1}{\sqrt[7]{\cos x}} &< \text{Write the function as a power:} \\
&= \frac{d}{dx} (\cos x)^{-\frac{1}{7}} &< \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x)
\end{aligned}$$

$$= -\frac{1}{7}(\cos x)^{-\frac{1}{7}-1} \frac{d}{dx} \cos x \quad \blacktriangleleft \text{Simplify and apply: } \frac{d}{dx} \cos x = -\sin x$$

$$= -\frac{1}{7}(\cos x)^{-\frac{8}{7}} (-\sin x) \quad \blacktriangleleft \text{Simplify:}$$

$$\therefore \frac{d}{dx} \frac{1}{\sqrt[7]{\cos x}} = \frac{\sin x}{7\sqrt[7]{\cos^8 x}}$$

$$6. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \sin^7 x \quad \blacktriangleleft \text{Write the function as a power:}$$

$$= \frac{d}{dx} (\sin x)^7 \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x)$$

$$= 7(\sin x)^{7-1} \frac{d}{dx} \sin x \quad \blacktriangleleft \text{Simplify and apply: } \frac{d}{dx} \sin x = \cos x$$

$$= 7(\sin x)^6 \cos x \quad \blacktriangleleft \text{Simplify:}$$

$$\therefore \frac{d}{dx} \sin^7 x = 7 \cos x \sin^6 x$$

$$7. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \cos^6 x \quad \blacktriangleleft \text{Write the function as a power:}$$

$$= \frac{d}{dx} (\cos x)^6 \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x)$$

$$= 6(\cos x)^{6-1} \frac{d}{dx} \cos x \quad \blacktriangleleft \text{Simplify and apply: } \frac{d}{dx} \cos x = -\sin x$$

$$= 6(\cos x)^5 (-\sin x) \quad \blacktriangleleft \text{Simplify:}$$

$$\therefore \frac{d}{dx} \cos^6 x = -6 \sin x \cos^5 x$$

$$8. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \frac{1}{\ln^3 x} \quad \blacktriangleleft \text{Write the function as a power:}$$

$$= \frac{d}{dx} (\ln x)^{-3} \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x)$$

$$= -3(\ln x)^{-3-1} \frac{d}{dx} \ln x \quad \blacktriangleleft \text{Simplify and apply: } \frac{d}{dx} \ln x = \frac{1}{x}$$

$$= -3(\ln x)^{-4} \frac{1}{x} \quad \blacktriangleleft \text{Simplify:}$$

$$\therefore \frac{d}{dx} \frac{1}{\ln^3 x} = \frac{-3}{x \ln^4 x}$$

$$9. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} (4^x)^6 \quad \blacktriangleleft \text{Write the function as a power:}$$

$$= \frac{d}{dx} (4^x)^6 \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x)$$

$$= 6(4^x)^{6-1} \frac{d}{dx} (4^x) \quad \blacktriangleleft \text{Simplify and apply: } \frac{d}{dx} b^x = (\ln b)b^x$$

$$= 6(4^x)^5 (\ln 4)(4^x) \quad \blacktriangleleft \text{Simplify:}$$

$$\therefore \frac{d}{dx} (4^x)^6 = 6(\ln 4)4^{6x}$$

$$10. f'(x) = \frac{d}{dx} f(x) = \frac{d}{dx} \frac{1}{(e^x)^8} \quad \blacktriangleleft \text{Write the function as a power:}$$

$$= \frac{d}{dx} (e^x)^{-8} \quad \blacktriangleleft \text{Apply: } \frac{d}{dx} (f(x))^n = n(f(x))^{n-1} \frac{d}{dx} f(x)$$

$$= -8(e^x)^{-8-1} \frac{d}{dx} e^x \quad \blacktriangleleft \text{Simplify and apply: } \frac{d}{dx} e^x = e^x$$

$$= -8(e^x)^{-9} e^x \quad \blacktriangleleft \text{Simplify:}$$

$$\therefore \frac{d}{dx} \frac{1}{(e^x)^8} = \frac{-8}{e^{8x}}$$