

1. Differentiate:  $f(x) = \frac{x^2}{\log x}$

2. Differentiate:  $f(x) = \frac{\cos x}{5^x}$

3. Differentiate:  $f(x) = \frac{\log x}{5^x}$

4. Differentiate:  $f(x) = \frac{10^x}{\cos x}$

5. Differentiate:  $f(x) = \frac{\ln x}{\cos x}$

6. Differentiate:  $f(x) = \frac{10^x}{\sqrt{x}}$

7. Differentiate:  $f(x) = \frac{\cos x}{e^x}$

8. Differentiate:  $f(x) = \frac{\log x}{\sqrt{x}}$

9. Differentiate:  $f(x) = \frac{\cos x}{\ln x}$

10. Differentiate:  $f(x) = \frac{10^x}{4^x}$

ANSWERS:

1.  $f'(x) = \frac{x \log x - \log x^2}{x^2}$

2.  $f'(x) = \frac{x^5 \cos x - 5x^4 \sin x}{x^{10}}$

3.  $f'(x) = \frac{x^5 \log x - \log x^5}{x^6}$

4.  $f'(x) = \frac{x^2 \cos x - \log x^2 \sin x}{x^4}$

5.  $f'(x) = \frac{x^2 \cos x + x \sin x}{x^3}$

6.  $f'(x) = \frac{x \log x - x^2 \sqrt{x}}{x^3}$

7.  $f'(x) = \frac{x^e \cos x - x \sin x}{x^2}$

8.  $f'(x) = \frac{x \log x - x \sqrt{x}}{x^2}$

9.  $f'(x) = \frac{x \log x - \cos x}{x^2}$

10.  $f'(x) = \frac{x^4 \log x - 4x^3}{x^8}$

Solutions:

$$\begin{aligned}
 1. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{x^2}{\log x} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
 &= \frac{\left(\frac{d}{dx} x^2\right) \log x - x^2 \frac{d}{dx} \log x}{(\log x)^2} && \blacktriangleleft \text{Apply: } \frac{d}{dx} x^2 = 2x \quad \frac{d}{dx} \log x = \frac{1}{(\ln 10)x} \\
 &= \frac{(2x) \log x - x^2 \left(\frac{1}{(\ln 10)x}\right)}{(\log x)^2} && \blacktriangleleft \text{Simplify, if necessary.} \\
 \therefore \frac{d}{dx} \frac{x^2}{\log x} &= \frac{2x \log x - \frac{x}{\ln 10}}{\log^2 x}
 \end{aligned}$$

$$\begin{aligned}
 2. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{\cos x}{5^x} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
 &= \frac{\left(\frac{d}{dx} \cos x\right) 5^x - \cos x \frac{d}{dx} 5^x}{(5^x)^2} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \cos x = -\sin x \quad \frac{d}{dx} b^x = (\ln b)b^x \\
 &= \frac{(-\sin x) 5^x - \cos x ((\ln 5)(5^x))}{(5^x)^2} && \blacktriangleleft \text{Simplify, if necessary.} \\
 \therefore \frac{d}{dx} \frac{\cos x}{5^x} &= \frac{-\sin x - (\ln 5) \cos x}{5^x}
 \end{aligned}$$

$$\begin{aligned}
 3. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{\log x}{5^x} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
 &= \frac{\left(\frac{d}{dx} \log x\right) 5^x - \log x \frac{d}{dx} 5^x}{(5^x)^2} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \log x = \frac{1}{(\ln 10)x} \quad \frac{d}{dx} b^x = (\ln b)b^x \\
 &= \frac{\left(\frac{1}{(\ln 10)x}\right) 5^x - \log x ((\ln 5)(5^x))}{(5^x)^2} && \blacktriangleleft \text{Simplify, if necessary.} \\
 \therefore \frac{d}{dx} \frac{\log x}{5^x} &= \frac{\frac{1}{(\ln 10)x} - (\ln 5) \log x}{5^x}
 \end{aligned}$$

$$\begin{aligned}
 4. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{10^x}{\cos x} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
 &= \frac{\left(\frac{d}{dx} 10^x\right) \cos x - 10^x \frac{d}{dx} \cos x}{(\cos x)^2} && \blacktriangleleft \text{Apply: } \frac{d}{dx} 10^x = (\ln 10)10^x \quad \frac{d}{dx} \cos x = -\sin x \\
 &= \frac{((\ln 10)(10^x)) \cos x - 10^x (-\sin x)}{(\cos x)^2} && \blacktriangleleft \text{Simplify, if necessary.} \\
 \therefore \frac{d}{dx} \frac{10^x}{\cos x} &= \frac{(\ln 10)(10^x) \cos x + \sin x 10^x}{\cos^2 x}
 \end{aligned}$$

$$\begin{aligned}
 5. \quad f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{\ln x}{\cos x} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} = \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
 &= \frac{\left(\frac{d}{dx} \ln x\right) \cos x - \ln x \frac{d}{dx} \cos x}{(\cos x)^2} && \blacktriangleleft \text{Apply: } \frac{d}{dx} \ln x = \frac{1}{x} \quad \frac{d}{dx} \cos x = -\sin x \\
 &= \frac{\left(\frac{1}{x}\right) \cos x - \ln x (-\sin x)}{(\cos x)^2} && \blacktriangleleft \text{Simplify, if necessary.} \\
 \therefore \frac{d}{dx} \frac{\ln x}{\cos x} &= \frac{\frac{1}{x} \cos x + \sin x \ln x}{\cos^2 x}
 \end{aligned}$$

$$\begin{aligned}
6. f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{10^x}{\sqrt{x}} &< \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} &= \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
&= \frac{\left(\frac{d}{dx} 10^x\right) \sqrt{x} - 10^x \frac{d}{dx} \sqrt{x}}{(\sqrt{x})^2} &< \text{Apply: } \frac{d}{dx} 10^x &= (\ln 10)10^x \quad \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}} \\
&= \frac{((\ln 10)(10^x)) \sqrt{x} - 10^x \left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2} &< \text{Simplify, if necessary.} \\
\therefore \frac{d}{dx} \frac{10^x}{\sqrt{x}} &= \frac{(\ln 10)(10^x)\sqrt{x} - \frac{1}{2\sqrt{x}}10^x}{x}
\end{aligned}$$

$$\begin{aligned}
7. f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{\cos x}{e^x} &< \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} &= \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
&= \frac{\left(\frac{d}{dx} \cos x\right) e^x - \cos x \frac{d}{dx} e^x}{(e^x)^2} &< \text{Apply: } \frac{d}{dx} \cos x &= -\sin x \quad \frac{d}{dx} e^x = e^x \\
&= \frac{(-\sin x) e^x - \cos x (e^x)}{(e^x)^2} &< \text{Simplify, if necessary.} \\
\therefore \frac{d}{dx} \frac{\cos x}{e^x} &= \frac{-\sin x - \cos x}{e^x}
\end{aligned}$$

$$\begin{aligned}
8. f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{\log x}{\sqrt{x}} &< \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} &= \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
&= \frac{\left(\frac{d}{dx} \log x\right) \sqrt{x} - \log x \frac{d}{dx} \sqrt{x}}{(\sqrt{x})^2} &< \text{Apply: } \frac{d}{dx} \log x &= \frac{1}{(\ln 10)x} \quad \frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}} \\
&= \frac{\left(\frac{1}{(\ln 10)x}\right) \sqrt{x} - \log x \left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2} &< \text{Simplify, if necessary.} \\
\therefore \frac{d}{dx} \frac{\log x}{\sqrt{x}} &= \frac{\frac{1}{(\ln 10)x} \sqrt{x} - \frac{1}{2\sqrt{x}} \log x}{x}
\end{aligned}$$

$$\begin{aligned}
9. f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{\cos x}{\ln x} &< \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} &= \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
&= \frac{\left(\frac{d}{dx} \cos x\right) \ln x - \cos x \frac{d}{dx} \ln x}{(\ln x)^2} &< \text{Apply: } \frac{d}{dx} \cos x &= -\sin x \quad \frac{d}{dx} \ln x = \frac{1}{x} \\
&= \frac{(-\sin x) \ln x - \cos x \left(\frac{1}{x}\right)}{(\ln x)^2} &< \text{Simplify, if necessary.} \\
\therefore \frac{d}{dx} \frac{\cos x}{\ln x} &= \frac{-\sin x \ln x - \frac{1}{x} \cos x}{\ln^2 x}
\end{aligned}$$

$$\begin{aligned}
10. f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \frac{10^x}{4^x} &< \text{Apply: } \frac{d}{dx} \frac{f(x)}{g(x)} &= \frac{\left(\frac{d}{dx} f(x)\right) g(x) - f(x) \frac{d}{dx} g(x)}{g^2(x)} \\
&= \frac{\left(\frac{d}{dx} 10^x\right) 4^x - 10^x \frac{d}{dx} 4^x}{(4^x)^2} &< \text{Apply: } \frac{d}{dx} 10^x &= (\ln 10)10^x \quad \frac{d}{dx} b^x = (\ln b)b^x \\
&= \frac{((\ln 10)(10^x)) 4^x - 10^x ((\ln 4)(4^x))}{(4^x)^2} &< \text{Simplify, if necessary.} \\
\therefore \frac{d}{dx} \frac{10^x}{4^x} &= \frac{(\ln 10)(10^x) - (\ln 4)10^x}{4^x}
\end{aligned}$$

