| MCV4U, CALCULUS AND VECTORS<br>**TEST Chapter 2**<br>**DIFFERENTIATION RULES**<br>Teacher: Teodoru GUGOIU | Date …….....<br>Name …….....| $\frac{\quad}{50}$ = |
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**Questions 1-5 are Multiple-Choice questions** [K/U 1 mark each]

**1.** Which of the following relations is INCORRECT

A) $(x^3)' = 3x^2$ B) $(2\sqrt{x})' = x^{-1/2}$ C) $\left(\frac{1}{x^2}\right)' = \frac{-1}{x^3}$ D) $\left(\frac{1}{\sqrt[3]{x}}\right)' = \frac{-1}{3x\sqrt[3]{x}}$

**2.** The equation of the tangent line to the curve $y = x^2$ at the point $P(1,1)$ is

A) $y = -x + 2$ B) $y = 2x - 1$ C) $y = -2x - 1$ D) $y = x + 1$

**3.** The derivative function of the function $f(x) = \frac{x-1}{x+1}$ is

A) $f'(x) = \frac{-2x}{x^2+1}$ B) $f'(x) = \frac{-x}{(x+1)^2}$ C) $f'(x) = \frac{2}{(x+1)^2}$ D) $f'(x) = \frac{-2x}{(x+1)^2}$

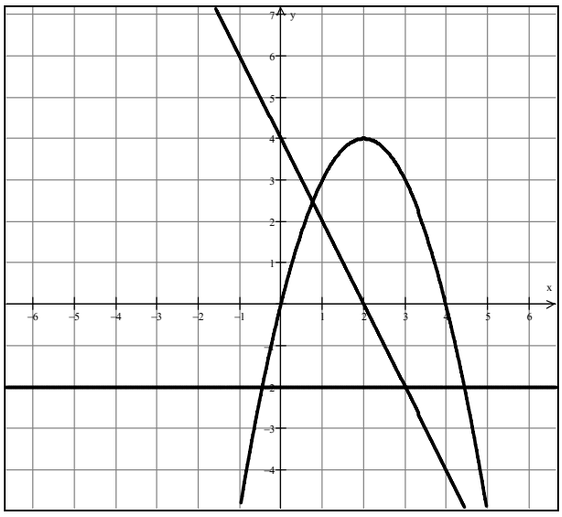
**4.** A function $f(x) = \frac{1}{x}$ is not differentiable at $x = 0$ because:

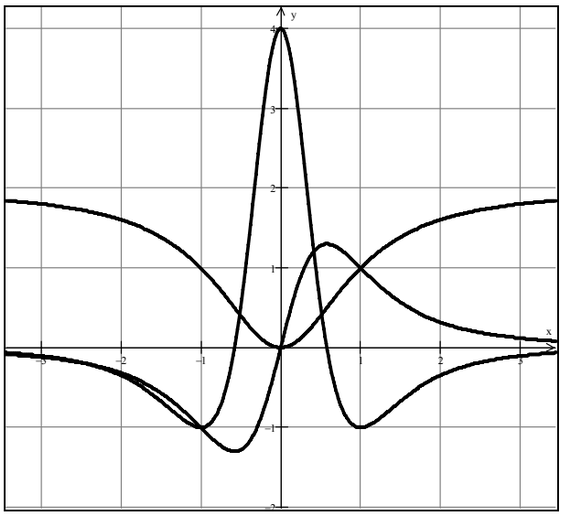
A) function $f$ has a vertical tangent at $x = 0$
B) function $f$ has a removable discontinuity at $x = 0$
C) function $f$ is not defined at $x = 0$
D) function $f$ has a horizontal tangent at $x = 0$

**5.** If $f'(x) = -3x^2 - 2x$ then a possible function $f(x)$ is

A) $f(x) = x^3 - x^2 + x$ B) $f(x) = -6x - 2$ C) $f(x) = -x^3 - x^2$ D) $f(x) = -x^3 + x^2 + x$

**6.** For each case, identify the function $f$ and its derivatives $f'$ and $f''$. [K/U 3 marks]




**Questions 7-15 a long answer questions. Show your work.**

**7.** At what points does the curve $y = 2x^3 + 3x^2 - 12x - 1$ have a horizontal tangent? [A 3 marks]

**8.** For each case, find $f'(x)$. [K/U 6 marks]

[2] a) $f(x) = 1 - x^3 + \frac{2}{x^4}$

[2] b) $f(x) = (x^2 + x)(x^3 - 1)$

[2] c) $f(x) = (\sqrt{x} - 2x)^5$

**9.** For each case, find $f'(x)$, $f''(x)$, and $f'''(x)$. [K/U 6 marks]

[3] a) $f(x) = x^4 - 2x^3$

[3] b) $f(x) = \frac{3x}{x+1}$

**10.** Differentiate [A 3 marks]

$f(x) = x + \sqrt{x^2 + \sqrt{x^2 - x}}$

**11.** Show that there are no tangents to the curve $y = \frac{x}{x-1}$ with positive slope. What can be concluded about the graph? [A 3 marks]

**12.** Find a quadratic function in the form $f(x) = ax^2 + bx + c$ satisfying the following conditions: $f(1) = -1$, $f'(1) = -3$, and $f''(1) = -6$. [A 3 marks]

**13.** For each case, explain where the function is not differentiable and why. [AC 3 marks]

[1.5] a) $f(x) = \sqrt[3]{x}$

[1.5] b) $f(x) = |x^2 - 3x + 2|$

**14.** Use the first principles to find the derivative of the following function. [A 3 marks]

$f(x) = -3x + \frac{2}{x^2}$

**15.** Find the equation of the tangent line to the curve $f(x) = \sqrt[3]{(x+1)^2}$ at the point $P(0,1)$. Graph the function $f$ and the tangent line. [A 3 marks]

**16.** A position function of a particle is given by $s(t) = 2t^3 - 6t$ . [A 9 marks]

[1] a) Find the moments of time when the particle is in origin.

[1.5] b) Find the velocity function and the moments of time when the particle is at rest.

[1.5] c) Find the acceleration function and the moments of time when the acceleration is zero.

[1] d) Find intervals of time when the particle is speeding up.

[2] Find the displacement and the total distance travelled over the interval $[-1,2]$ .

[2] d) Sketch the graph of $s(t)$ , $v(t)$ , and $a(t)$ on the grid provided bellow