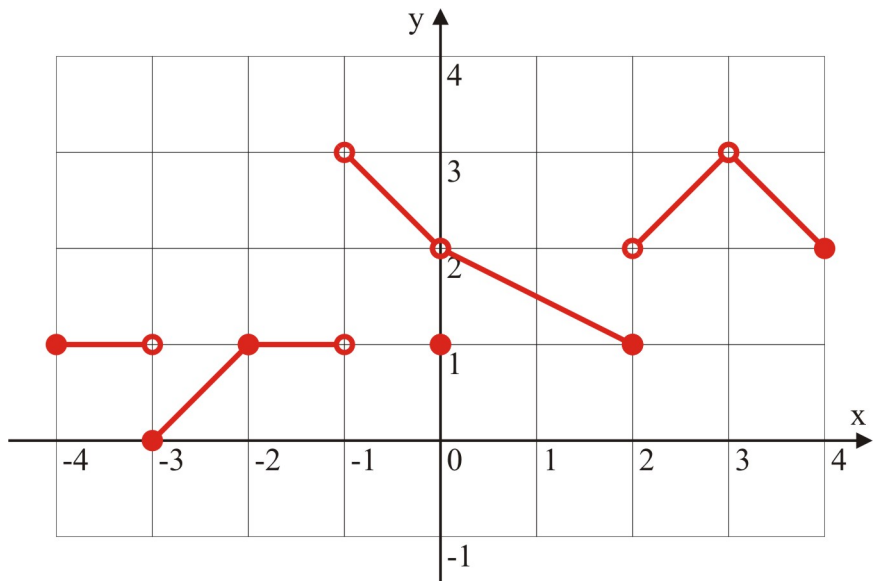


MCV4U, CALCULUS AND VECTORS <b>TEST Chapter 1</b> <b>LIMITS</b> Teacher: Teodoru Gugoiu	Date ..... Name .....	$\frac{\quad}{45} =$
--	--------------------------	----------------------

1. The function  $f$  is defined by the graph represented in the right figure. Find: [3]

- a)  $\lim_{x \rightarrow 3} f(x)$
- b)  $\lim_{x \rightarrow 2^-} f(x)$
- c)  $\lim_{x \rightarrow -3^+} f(x)$
- d)  $\lim_{x \rightarrow -1} f(x)$
- e)  $\lim_{x \rightarrow 3} f(x)$
- f)  $\lim_{x \rightarrow -3} f(x)$



2. The function  $f$  is defined by the graph represented in the above figure. Analyse the continuity (continuous, discontinuous, type of discontinuity) of the function  $f$  at each number specified below. [3]

- a) at  $x = 2$
- b) at  $x = 3$
- c) at  $x = 1$

3. Consider the piecewise defined function  $f(x) = \begin{cases} x^2 - 2, & x < -2 \\ -x, & -2 \leq x \leq 0 \\ \sqrt{1+x}, & x > 0 \end{cases}$  [3]

Find:

- a)  $\lim_{x \rightarrow -2} f(x)$
- b)  $\lim_{x \rightarrow -1} f(x)$
- c)  $\lim_{x \rightarrow 0} f(x)$

4. Consider the piecewise defined function. Analyse the continuity of this function. [3]

$$f(x) = \begin{cases} x+1, & x < -1 \\ x^2, & -1 \leq x \leq 1 \\ \sqrt{x}, & x > 1 \end{cases}$$

5. Given  $\lim_{x \rightarrow 2} f(x) = 2$  and  $\lim_{x \rightarrow 2} g(x) = -4$ , use the limits laws to evaluate:

[3]

[1] a)  $\lim_{x \rightarrow 2} [f(x) - g(x)]$

[1] b)  $\lim_{x \rightarrow 2} [f(x)g(x)]$

[1] c)  $\lim_{x \rightarrow 2} [1 - f(x) + 2g(x)]$

6. Find each limit.

[6]

a)  $\lim_{x \rightarrow -3} \frac{x+3}{x^2-9}$

b)  $\lim_{x \rightarrow 2} \frac{x^2-x-2}{x-2}$

c)  $\lim_{x \rightarrow 1} \frac{1-\sqrt{x}}{x-1}$

7. Redefine the function  $f$  to remove the discontinuity and to make it continuous at any number.

[3]

$$f(x) = \frac{x^3 - 8}{x - 2}$$

8. Find the number  $a$  such that the function  $f$  is continuous at any number.

[3]

$$f(x) = \begin{cases} ax + \frac{4}{x}, & x < 2 \\ ax^2, & x \geq 2 \end{cases}$$

9. Consider the function

[5]

$$f(x) = \frac{2x+1}{2x-1}$$

[1] a) Find the average rate of change over the interval  $[1,3]$

[3] b) Find the instantaneous rate of change at  $x = 0$

[1] c) Find the equation of the tangent line at  $P(0,-1)$

10. A fireworks is launched vertically upward. Its position is described by the position function [4]

$h(t) = 30t - 2t^3$  (where  $h(t)$  is the altitude in meters at the time  $t$  in seconds). Find:

[1] a) The average velocity over the interval  $[1,3]$

[3] b) The instantaneous velocity at the moment  $t = 2$

11. Use technology (scientific calculator) to estimate the instantaneous rate of change at  $a = 1$  for the function

$$f(x) = \sqrt{2x^2 - \sqrt[3]{x}} \quad \text{using } h = 0.01 \text{ and } 0.000001. \quad [3]$$

12. Find the slope of the tangent line to the graph of the curve

$$f(x) = 2x + \sqrt{x} \quad \text{at the generic point } P(a, f(a)). \quad [3]$$

13. Evaluate the limit

[3]

$$\lim_{x \rightarrow 1} \frac{\sqrt[3]{x} - x}{\sqrt{x} - x}$$