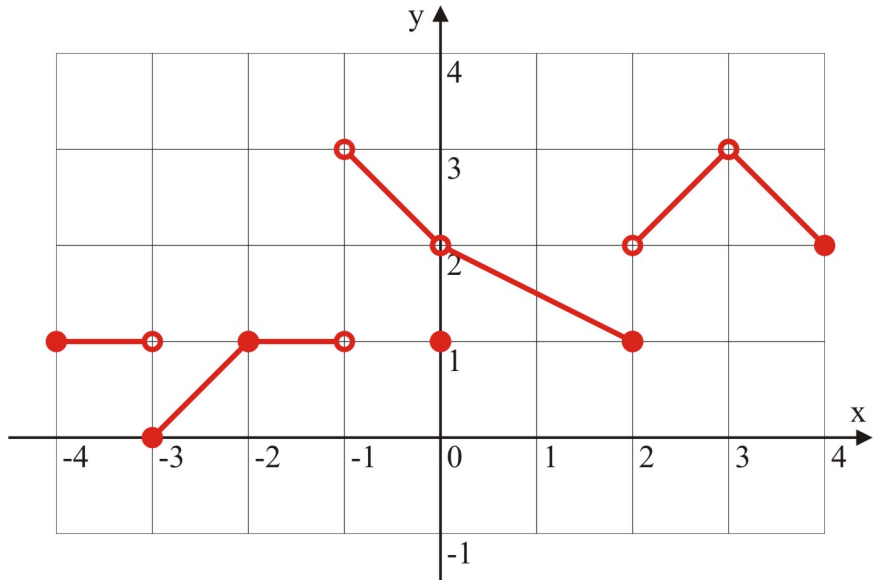


MCV4U, CALCULUS AND VECTORS TEST Chapter 1 LIMITS Teacher: Teodoru Gugoiu	Date Name	$\frac{\quad}{45} =$
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1. The function f is defined by the graph represented in the right figure. Find: [3]

- a) $\lim_{x \rightarrow 0} f(x)$
- b) $\lim_{x \rightarrow -3^-} f(x)$
- c) $\lim_{x \rightarrow 2^+} f(x)$
- d) $\lim_{x \rightarrow -1} f(x)$
- e) $\lim_{x \rightarrow 3} f(x)$
- f) $\lim_{x \rightarrow -3} f(x)$



2. The function f is defined by the graph represented in the above figure. Analyse the continuity (continuous, discontinuous, type of discontinuity) of the function f at each number specified below. [3]

- a) at $x = -1$
- b) at $x = 0$
- c) at $x = -2$

3. Consider the piecewise defined function $f(x) = \begin{cases} x+1, & x < -1 \\ x^2, & -1 \leq x \leq 1 \\ \sqrt{x}, & x > 1 \end{cases}$ [3]

Find:

- a) $\lim_{x \rightarrow -1} f(x)$
- b) $\lim_{x \rightarrow 0} f(x)$
- c) $\lim_{x \rightarrow 1} f(x)$

4. Consider the piecewise defined function. Analyse the continuity of this function. [3]

$$f(x) = \begin{cases} x^2 - 2, & x < -2 \\ -x, & -2 \leq x \leq 0 \\ \sqrt{1+x}, & x > 0 \end{cases}$$

5. Given $\lim_{x \rightarrow 2} f(x) = 2$ and $\lim_{x \rightarrow 2} g(x) = -4$, use the limits laws to evaluate: [3]

[1] a) $\lim_{x \rightarrow 2} [f(x) + g(x)]$

[1] b) $\lim_{x \rightarrow 2} [f(x) / g(x)]$

[1] c) $\lim_{x \rightarrow 2} [3f(x) - 2g(x)]$

6. Find each limit. [6]

a) $\lim_{x \rightarrow -2} \frac{x^2 - 4}{x + 2}$

b) $\lim_{x \rightarrow 1} \frac{x - 1}{x^2 - 3x + 2}$

c) $\lim_{x \rightarrow 4} \frac{\sqrt{x} - 2}{x - 4}$

7. Redefine the function f to remove the discontinuity and to make it continuous at any number. [3]

$$f(x) = \frac{x^3 + 1}{x + 1}$$

8. Find the number a such that the function f is continuous at any number. [3]

$$f(x) = \begin{cases} 2x^2 - ax, & x < 1 \\ a + \sqrt{x-1}, & x \geq 1 \end{cases}$$

9. Consider the function

[5]

$$f(x) = \frac{x-1}{x+1}$$

[1] a) Find the average rate of change over the interval $[1,3]$

[3] b) Find the instantaneous rate of change at $x = 0$

[1] c) Find the equation of the tangent line at $P(0,-1)$

10. A fireworks is launched vertically upward. Its position is described by the position function

[4]

$h(t) = 20t - t^3$ (where $h(t)$ is the altitude in meters at the time t in seconds). Find:

[1] a) The average velocity over the interval $[1,3]$

[3] b) The instantaneous velocity at the moment $t = 2$

11. Use technology (scientific calculator) to estimate the instantaneous rate of change at $a = 1$ for the function

$$f(x) = \sqrt[3]{2x^2 - \sqrt{x}} \quad \text{using } h = 0.01 \text{ and } 0.000001. \quad [3]$$

12. Find the slope of the tangent line to the graph of the curve $f(x) = x + \frac{2}{x}$ at the generic point $P(a, f(a))$. [3]

13. Evaluate the limit [3]

$$\lim_{x \rightarrow 1} \frac{\sqrt{x} - 1}{1 - \sqrt[3]{x}}$$