

MCV4U Quiz #4 Exponential and Logarithmic Functions

Name

1. Differentiate.

a) $f(x) = \log_{10}(3x+1)^4$

$$f'(x) = \frac{4(3x+1)^3 (3)}{(3x+1)^4 \ln 10} = \frac{12}{(\ln 10)(3x+1)}$$

b) $f(x) = \sqrt{x+e^{1-x^2}}$

$$f'(x) = \frac{1 + e^{1-x^2}(-2x)}{2\sqrt{x+e^{1-x^2}}} = \frac{1-2xe^{1-x^2}}{2\sqrt{x+e^{1-x^2}}}$$

c) $f(x) = (x - \ln x)(2x^2)$

$$\begin{aligned} f'(x) &= \left(1 - \frac{1}{x}\right) 2x^2 + (x - \ln x) 2x^2 (2x) (\ln 2) \\ &= 2x^2 \left[1 - \frac{1}{x} + 2(\ln 2)x^2 - 2(\ln 2)x(\ln x)\right] \end{aligned}$$

2. Find the intervals of increase and decrease for $f(x) = x^2 e^{-x}$.

$$f'(x) = 2x e^{-x} + x^2 e^{-x} (-1)$$

$$= x e^{-x} (2 - x)$$

$$f'(x) = 0 \text{ at } x=0 \text{ or } x=2$$

x		0	2		
f'(x)	-	0	+	0	-
f(x)		↘	↗		↘

f increases over $(0, 2)$
 decreases over $(-\infty, 0)$ or $(2, \infty)$

3. Find the intervals of concavity for $f(x) = x \ln(x^2)$.

$$f'(x) = \ln(x^2) + x \cdot \frac{2x}{x^2}$$

$$= \ln(x^2) + 2 \quad \Rightarrow \quad x^2 = e^{-2} \Rightarrow x = \pm e^{-1}$$

$$f''(x) = \frac{2x}{x^2} = \frac{2}{x}$$

$f''(x) = 0$ no solution

x			-	0		
$f''(x)$	-	-	-		+	+
$f(x)$	↪	↪	↪	↻	↻	↻

f is concave upward over $(-\infty, \infty)$ or $(0, \infty)$
 downward over $(-\infty, 0)$

