SOLUTIONS

MCV4U Quiz #4 Exponential and Logarithmic Functions Name ....

1. Differentiate:

a) $f(x)=\ln\frac{x}{\sqrt{x+1}}$

$$f'(x) = \frac{\sqrt{x+1}}{x} \cdot \frac{\sqrt{x+1} - x\frac{1}{2\sqrt{x+1}}}{x+1} = \frac{2x+2-x}{2x(x+1)} = \frac{x+2}{2x(x+1)}$$

$$= \frac{\cancel{2(x+1)}}{2x\cancel{(x+1)}} - \frac{\cancel{x}}{2\cancel{x}(x+1)} = \frac{1}{x} - \frac{1}{2(x+1)}$$

b) $f(x)=(1+5e^{-3x})^4$

$$f'(x) = 4(1+5e^{-3x})^3 \cdot (5e^{-3x})(-3) = -60e^{-3x}(1+5e^{-3x})^3$$

c) $f(x)=(x-\ln x)(2^{x^2})$

$$f'(x) = \left(1-\frac{1}{x}\right)2^{x^2} + (x-\ln x)\,2^{x^2}(\ln 2)(2x)$$

$$= 2^{x^2}\left[1-\frac{1}{x} + 2x^2(\ln 2) - 2(\ln 2)(\ln x)\,x\right]$$

2. Find the local minimum or maximum points for the function $f(x)=x\ln(x^2)$.

$$f'(x) = \ln x^2 + x\,\frac{2x}{x^2} = \ln x^2 + 2$$

$f'(x)=0$ at $x^2 = e^{-2} \iff x = \pm\frac{1}{e}$

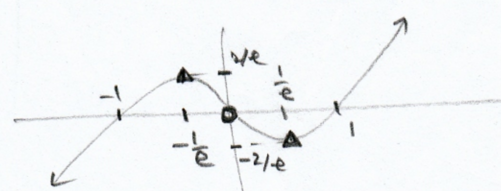
x	-1	$-\frac{1}{e}$		$\frac{1}{e}$	1
$f'(x)$	$+$	0	$-$	0	$+$
$f(x)$	$\nearrow$	$\frac{2}{e}$ LM	$\searrow$	$-\frac{2}{e}$ Lm	$\nearrow$

$f(-e^{-1}) = -e^{-1}\ln(e^{-2}) = \frac{2}{e}$

$f(e^{-1}) = e^{-1}\ln(e^{-2}) = -\frac{2}{e}$

$\therefore$ LM: $\left(-\frac{1}{e}, \frac{2}{e}\right)$

Lm: $\left(+\frac{1}{e}, -\frac{2}{e}\right)$



3. Find the inflection points for the function $f(x) = x^2 e^{-x}$.

$f'(x) = 2x e^{-x} + x^2 e^{-x}(-1)$

$= x e^{-x}(2-x) = 2x e^{-x} - x^2 e^{-x}$

$f''(x) = 2e^{-x} + 2x e^{-x}(-1) - 2x e^{-x} - x^2 e^{-x}(-1)$

$= e^{-x}\left[2 - 2x - 2x + x^2\right]$

$= e^{-x}\left[x^2 - 4x + 2\right]$

$f''(x) = 0$ at $x = \frac{4 \pm \sqrt{16-8}}{2} = 2 \pm \sqrt{2}$

x		$2-\sqrt{2}$		$2+\sqrt{2}$	
$f''(x)$	$+$	0	$-$	0	$+$
$f(x)$		IP		IP	

$A\left(2-\sqrt{2},\ (2-\sqrt{2})^2 e^{-(2-\sqrt{2})}\right) \approx (0.586,\ 0.191)$

$B\left(2+\sqrt{2},\ (2+\sqrt{2})^2 e^{-(2+\sqrt{2})}\right) \simeq (3.414,\ 0.384)$