1. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = -x^2 + 2x^3$

2. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = -2 + x$

3. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = 1 - 2x$

4. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = -1 + 2x^4$

5. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = -x + 3x^4$

6. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = -x + 3x^2$

7. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = 3x - x^4$

8. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = -2x + 2x^4$

9. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = x^3 + 2x^4$

10. Use the first principles formula $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$ to find the derivative function for the folowing function: $f(x) = 2x + 3x^2$

Answers:

1. $-2x + 6x^2$ 2. 1 3. -2 4. $8x^3$ 5. $-1 + 12x^3$ 6. $-1 + 6x$
7. $3 - 4x^3$ 8. $-2 + 8x^3$ 9. $3x^2 + 8x^3$ 10. $2 + 6x$

Solutions:

First, we need to know the following algebraic identities:

$a^2 - b^2 = (a-b)(a+b)$

$a^3 - b^3 = (a-b)(a^2+ab+b^2)$

$a^3 + b^3 = (a+b)(a^2-ab+b^2)$

$a^4 - b^4 = (a-b)(a+b)(a^2+b^2)$

Second, we need to understand the computation of the following fundamental limits:

$$L_0 = \lim_{h\to 0} \frac{1-1}{h} = \lim_{h\to 0} \frac{0}{h} = \lim_{h\to 0} 0 = 0$$

$$L_1 = \lim_{h\to 0} \frac{(x+h)-x}{h} = \lim_{h\to 0} \frac{h}{h} = \lim_{h\to 0} 1 = 1$$

$$L_2 = \lim_{h\to 0} \frac{(x+h)^2-x^2}{h} = \lim_{h\to 0} \frac{[(x+h)-x][(x+h)+x]}{h} = \lim_{h\to 0} \frac{h(2x+h)}{h} = \lim_{h\to 0} 2x+h = 2x$$

$$L_3 = \lim_{h\to 0} \frac{(x+h)^3-x^3}{h} = \lim_{h\to 0} \frac{[(x+h)-x][(x+h)^2+(x+h)x+x^2]}{h}$$
$$= \lim_{h\to 0} \frac{h[(x+h)^2+(x+h)x+x^2]}{h} = \lim_{h\to 0} (x+h)^2+(x+h)x+x^2 = 3x^2$$

$$L_4 = \lim_{h\to 0} \frac{(x+h)^4-x^4}{h} = \lim_{h\to 0} \frac{[(x+h)-x][(x+h)+x][(x+h)^2+x^2]}{h}$$
$$= \lim_{h\to 0} \frac{h[(x+h)+x][(x+h)^2+x^2]}{h} = \lim_{h\to 0} [(x+h)+x][(x+h)^2+x^2] = (2x)(2x^2) = 4x^3$$

1. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ ► Use substitution:

$= \lim_{h\to 0} \dfrac{[-(x+h)^2+2(x+h)^3]-[-x^2+2x^3]}{h}$ ► Group the like terms and factor:

$= -\lim_{h\to 0} \dfrac{(x+h)^2-x^2}{h} + 2\lim_{h\to 0} \dfrac{(x+h)^3-x^3}{h}$ ► Identify the fundamental limits:

$= -L_2 + 2L_3$ ► Substitute the values of the fundamental limits:

$= -(2x) + 2(3x^2)$ ► Simplify:

$= -2x + 6x^2$

2. $f'(x) = \lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$ ► Use substitution:

$= \lim_{h\to 0} \dfrac{[-2+(x+h)]-[-2+x]}{h}$ ► Group the like terms and factor:

$= -2\lim_{h\to 0} \dfrac{1-1}{h} + \lim_{h\to 0} \dfrac{(x+h)-x}{h}$ ► Identify the fundamental limits:

$= -2L_0 + L_1$ ► Substitute the values of the fundamental limits:

$= -2(0) + (1)$ ► Simplify:

$= 1$

3. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h\to 0} \frac{[1 - 2(x+h)] - [1 - 2x]}{h}$ ▶ Group the like terms and factor:

$= \lim_{h\to 0} \frac{1-1}{h} - 2 \lim_{h\to 0} \frac{(x+h) - x}{h}$ ▶ Identify the fundamental limits:

$= L_0 - 2L_1$ ▶ Substitute the values of the fundamental limits:

$= (0) - 2(1)$ ▶ Simplify:

$= -2$

4. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h\to 0} \frac{[-1 + 2(x+h)^4] - [-1 + 2x^4]}{h}$ ▶ Group the like terms and factor:

$= -\lim_{h\to 0} \frac{1-1}{h} + 2 \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h}$ ▶ Identify the fundamental limits:

$= -L_0 + 2L_4$ ▶ Substitute the values of the fundamental limits:

$= -(0) + 2(4x^3)$ ▶ Simplify:

$= 8x^3$

5. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h\to 0} \frac{[-(x+h) + 3(x+h)^4] - [-x + 3x^4]}{h}$ ▶ Group the like terms and factor:

$= -\lim_{h\to 0} \frac{(x+h) - x}{h} + 3 \lim_{h\to 0} \frac{(x+h)^4 - x^4}{h}$ ▶ Identify the fundamental limits:

$= -L_1 + 3L_4$ ▶ Substitute the values of the fundamental limits:

$= -(1) + 3(4x^3)$ ▶ Simplify:

$= -1 + 12x^3$

6. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h\to 0} \frac{[-(x+h) + 3(x+h)^2] - [-x + 3x^2]}{h}$ ▶ Group the like terms and factor:

$= -\lim_{h\to 0} \frac{(x+h) - x}{h} + 3 \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$ ▶ Identify the fundamental limits:

$= -L_1 + 3L_2$ ▶ Substitute the values of the fundamental limits:

$= -(1) + 3(2x)$ ▶ Simplify:

$= -1 + 6x$

7. $f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h\to 0} \frac{[3(x+h) - (x+h)^4] - [3x - x^4]}{h}$ ▶ Group the like terms and factor:

$= 3 \lim_{h \to 0} \frac{(x+h) - x}{h} - \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$ ▶ Identify the fundamental limits:

$= 3L_1 - L_4$ ▶ Substitute the values of the fundamental limits:

$= 3(1) - (4x^3)$ ▶ Simplify:

$= 3 - 4x^3$

8. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h \to 0} \frac{[-2(x+h) + 2(x+h)^4] - [-2x + 2x^4]}{h}$ ▶ Group the like terms and factor:

$= -2 \lim_{h \to 0} \frac{(x+h) - x}{h} + 2 \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$ ▶ Identify the fundamental limits:

$= -2L_1 + 2L_4$ ▶ Substitute the values of the fundamental limits:

$= -2(1) + 2(4x^3)$ ▶ Simplify:

$= -2 + 8x^3$

9. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h \to 0} \frac{[(x+h)^3 + 2(x+h)^4] - [x^3 + 2x^4]}{h}$ ▶ Group the like terms and factor:

$= \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h} + 2 \lim_{h \to 0} \frac{(x+h)^4 - x^4}{h}$ ▶ Identify the fundamental limits:

$= L_3 + 2L_4$ ▶ Substitute the values of the fundamental limits:

$= (3x^2) + 2(4x^3)$ ▶ Simplify:

$= 3x^2 + 8x^3$

10. $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$ ▶ Use substitution:

$= \lim_{h \to 0} \frac{[2(x+h) + 3(x+h)^2] - [2x + 3x^2]}{h}$ ▶ Group the like terms and factor:

$= 2 \lim_{h \to 0} \frac{(x+h) - x}{h} + 3 \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$ ▶ Identify the fundamental limits:

$= 2L_1 + 3L_2$ ▶ Substitute the values of the fundamental limits:

$= 2(1) + 3(2x)$ ▶ Simplify:

$= 2 + 6x$

