

1. Estimate the instantaneous rate of change of the function $f(x) = x - 3 + \frac{1}{x} + \frac{-2}{x^2}$ at the number $x = -3$ using:
 a) $h = 0.01$ b) $h = 0.0001$
2. Estimate the instantaneous rate of change of the function $f(x) = \frac{1 + 3x}{-1 - x}$ at the number $x = -2$ using:
 a) $h = 0.01$ b) $h = 0.0001$
3. Estimate the instantaneous rate of change of the function $f(x) = -3x + 1 + \frac{-3}{x} + \frac{-1}{x^2}$ at the number $x = -2$ using:
 a) $h = 0.01$ b) $h = 0.0001$
4. Estimate the instantaneous rate of change of the function $f(x) = -2 - 3x + 3x^2 - x^3$ at the number $x = 0$ using:
 a) $h = 0.01$ b) $h = 0.0001$
5. Estimate the instantaneous rate of change of the function $f(x) = -x + -2 + \frac{-3}{x} + \frac{-1}{x^2}$ at the number $x = -2$ using:
 a) $h = 0.01$ b) $h = 0.0001$
6. Estimate the instantaneous rate of change of the function $f(x) = 3 - 3x + x^2 + 3x^3$ at the number $x = -2$ using:
 a) $h = 0.01$ b) $h = 0.0001$
7. Estimate the instantaneous rate of change of the function $f(x) = \sqrt{-3 + x + x^2 - 2x^3}$ at the number $x = -2$ using:
 a) $h = 0.01$ b) $h = 0.0001$
8. Estimate the instantaneous rate of change of the function $f(x) = \sqrt{-1 - x^2 - 3x^3}$ at the number $x = -3$ using:
 a) $h = 0.01$ b) $h = 0.0001$
9. Estimate the instantaneous rate of change of the function $f(x) = 2x + -2 + \frac{-1}{x} + \frac{-2}{x^2}$ at the number $x = -1$ using:
 a) $h = 0.01$ b) $h = 0.0001$
10. Estimate the instantaneous rate of change of the function $f(x) = \sqrt{-2 - x - 2x^3}$ at the number $x = -3$ using:
 a) $h = 0.01$ b) $h = 0.0001$

- ANSWERS:
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|----|----|-------|----|-------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|-----|----|--------|----|--------|
| 1. | a) | 0.741 | b) | 0.741 | 2. | a) | -2.020 | b) | -2.000 | 3. | a) | -2.498 | b) | -2.500 | 4. | a) | 7. | a) | 28.998 | b) | 28.998 | 5. | a) | -3.000 | b) | -3.486 | 6. | a) | 28.830 | b) | 28.830 | 7. | a) | -3.485 | b) | -3.705 | 8. | a) | -4.447 | b) | -4.450 | 9. | a) | -1.051 | b) | -1.001 | 10. | a) | -3.708 | b) | -3.708 |
|----|----|-------|----|-------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|----|----|--------|----|--------|-----|----|--------|----|--------|

Solutions:

1.

$$\text{a) } h = 0.01 \quad x = -3 \quad f(x) = (-3) + -3 + \frac{1}{(-3)} + \frac{-2}{(-3)^2} = -6.55555555555556$$

$$f(x+h) = (-2.99) + -3 + \frac{1}{(-2.99)} + \frac{-2}{(-2.99)^2} = -6.5481593047058$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-6.5481593047058 - (-6.55555555555556)}{0.01} \approx 0.740$$

$$\text{b) } h = 0.0001 \quad x = -3 \quad f(x) = (-3) + -3 + \frac{1}{(-3)} + \frac{-2}{(-3)^2} = -6.55555555555556$$

$$f(x+h) = (-2.9999) + -3 + \frac{1}{(-2.9999)} + \frac{-2}{(-2.9999)^2} = -6.5554814825926$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-6.5554814825926 - (-6.55555555555556)}{0.0001} \approx 0.741$$

2.

$$\text{a) } h = 0.01 \quad x = -2 \quad f(x) = \frac{1+3(-2)}{-1-(-2)} = -5$$

$$f(x+h) = \frac{1+3(-1.99)}{-1-(-1.99)} = -5.020202020202$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-5.020202020202 - (-5)}{0.01} \approx -2.020$$

$$\text{b) } h = 0.0001 \quad x = -2 \quad f(x) = \frac{1+3(-2)}{-1-(-2)} = -5$$

$$f(x+h) = \frac{1+3(-1.9999)}{-1-(-1.9999)} = -5.000200020002$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-5.000200020002 - (-5)}{0.0001} \approx -2.000$$

3.

$$\text{a) } h = 0.01 \quad x = -2 \quad f(x) = -3(-2) + 1 + \frac{-3}{(-2)} + \frac{-1}{(-2)^2} = 8.25$$

$$f(x+h) = -3(-1.99) + 1 + \frac{-3}{(-1.99)} + \frac{-1}{(-1.99)^2} = 8.2250188126562$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{8.2250188126562 - (8.25)}{0.01} \approx -2.498$$

$$\text{b) } h = 0.0001 \quad x = -2 \quad f(x) = -3(-2) + 1 + \frac{-3}{(-2)} + \frac{-1}{(-2)^2} = 8.25$$

$$f(x+h) = -3(-1.9999) + 1 + \frac{-3}{(-1.9999)} + \frac{-1}{(-1.9999)^2} = 8.2497500018751$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{8.2497500018751 - (8.25)}{0.0001} \approx -2.500$$

4.

$$\text{a) } h = 0.01 \quad x = 0 \quad f(x) = -2 - 3(0) + 3(0)^2 - (0)^3 = -2$$

$$f(x+h) = -2 - 3(0.01) + 3(0.01)^2 - (0.01)^3 = -2.029701$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-2.029701 - (-2)}{0.01} \approx -2.970$$

$$\text{b) } h = 0.0001 \quad x = 0 \quad f(x) = -2 - 3(0) + 3(0)^2 - (0)^3 = -2$$

$$f(x+h) = -2 - 3(0.0001) + 3(0.0001)^2 - (0.0001)^3 = -2.000299970001$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-2.000299970001 - (-2)}{0.0001} \approx -3.000$$

5.

$$\text{a) } h = 0.01 \quad x = -2 \quad f(x) = -(-2) + -2 + \frac{-3}{(-2)} + \frac{-1}{(-2)^2} = 1.25$$

$$f(x+h) = -(-1.99) + -2 + \frac{-3}{(-1.99)} + \frac{-1}{(-1.99)^2} = 1.2450188126562$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{1.2450188126562 - (1.25)}{0.01} \approx -0.498$$

$$\text{b) } h = 0.0001 \quad x = -2 \quad f(x) = -(-2) + -2 + \frac{-3}{(-2)} + \frac{-1}{(-2)^2} = 1.25$$

$$f(x+h) = -(-1.9999) + -2 + \frac{-3}{(-1.9999)} + \frac{-1}{(-1.9999)^2} = 1.2499500018751$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{1.2499500018751 - (1.25)}{0.0001} \approx -0.500$$

6.

$$\text{a) } h = 0.01 \quad x = -2 \quad f(x) = 3 - 3(-2) + (-2)^2 + 3(-2)^3 = -11$$

$$f(x+h) = 3 - 3(-1.99) + (-1.99)^2 + 3(-1.99)^3 = -10.711697$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-10.711697 - (-11)}{0.01} \approx 28.830$$

$$\text{b) } h = 0.0001 \quad x = -2 \quad f(x) = 3 - 3(-2) + (-2)^2 + 3(-2)^3 = -11$$

$$f(x+h) = 3 - 3(-1.9999) + (-1.9999)^2 + 3(-1.9999)^3 = -10.997100169997$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{-10.997100169997 - (-11)}{0.0001} \approx 28.998$$

7.

$$\text{a) } h = 0.01 \quad x = -2 \quad f(x) = \sqrt{-3 + (-2) + (-2)^2 - 2(-2)^3} = 3.8729833462074$$

$$f(x+h) = \sqrt{-3 + (-1.99) + (-1.99)^2 - 2(-1.99)^3} = 3.8381373086433$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{3.8381373086433 - (3.8729833462074)}{0.01} \approx -3.485$$

$$\text{b) } h = 0.0001 \quad x = -2 \quad f(x) = \sqrt{-3 + (-2) + (-2)^2 - 2(-2)^3} = 3.8729833462074$$

$$f(x+h) = \sqrt{-3 + (-1.9999) + (-1.9999)^2 - 2(-1.9999)^3} = 3.8726347788034$$

$$IRC \approx \frac{f(x+h) - f(x)}{h} = \frac{3.8726347788034 - (3.8729833462074)}{0.0001} \approx -3.486$$

8.

$$\text{a) } h = 0.01 \quad x = -3 \quad f(x) = \sqrt{-1 - (-3)^2 - 3(-3)^3} = 8.4261497731764$$

$$f(x+h) = \sqrt{-1 - (-2.99)^2 - 3(-2.99)^3} = 8.3816822297198$$

$$IRC \cong \frac{f(x+h) - f(x)}{h} = \frac{8.3816822297198 - (8.4261497731764)}{0.01} \cong -4.447$$

$$\text{b) } h = 0.0001 \quad x = -3 \quad f(x) = \sqrt{-1 - (-3)^2 - 3(-3)^3} = 8.4261497731764$$

$$f(x+h) = \sqrt{-1 - (-2.9999)^2 - 3(-2.9999)^3} = 8.4257047337298$$

$$IRC \cong \frac{f(x+h) - f(x)}{h} = \frac{8.4257047337298 - (8.4261497731764)}{0.0001} \cong -4.450$$

9.

$$\text{a) } h = 0.01 \quad x = -1 \quad f(x) = 2(-1) + -2 + \frac{-1}{(-1)} + \frac{-2}{(-1)^2} = -5$$

$$f(x+h) = 2(-0.99) + -2 + \frac{-1}{(-0.99)} + \frac{-2}{(-0.99)^2} = -5.0105070911132$$

$$IRC \cong \frac{f(x+h) - f(x)}{h} = \frac{-5.0105070911132 - (-5)}{0.01} \cong -1.051$$

$$\text{b) } h = 0.0001 \quad x = -1 \quad f(x) = 2(-1) + -2 + \frac{-1}{(-1)} + \frac{-2}{(-1)^2} = -5$$

$$f(x+h) = 2(-0.9999) + -2 + \frac{-1}{(-0.9999)} + \frac{-2}{(-0.9999)^2} = -5.000100050007$$

$$IRC \cong \frac{f(x+h) - f(x)}{h} = \frac{-5.000100050007 - (-5)}{0.0001} \cong -1.001$$

10.

$$\text{a) } h = 0.01 \quad x = -3 \quad f(x) = \sqrt{-2 - (-3) - 2(-3)^3} = 7.4161984870957$$

$$f(x+h) = \sqrt{-2 - (-2.99) - 2(-2.99)^3} = 7.3791461565685$$

$$IRC \cong \frac{f(x+h) - f(x)}{h} = \frac{7.3791461565685 - (7.4161984870957)}{0.01} \cong -3.705$$

$$\text{b) } h = 0.0001 \quad x = -3 \quad f(x) = \sqrt{-2 - (-3) - 2(-3)^3} = 7.4161984870957$$

$$f(x+h) = \sqrt{-2 - (-2.9999) - 2(-2.9999)^3} = 7.4158276800367$$

$$IRC \cong \frac{f(x+h) - f(x)}{h} = \frac{7.4158276800367 - (7.4161984870957)}{0.0001} \cong -3.708$$