

1. Consider the following piece-wise defined function:  $f(x) = \begin{cases} 1 - 3x + 2x^2 & \text{if } x < -2 \\ 25 + 3x - x^2 & \text{if } x > -2 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow -2^-} f(x)$     b)  $\lim_{x \rightarrow -2^+} f(x)$     c)  $\lim_{x \rightarrow -2} f(x)$     d)  $\lim_{x \rightarrow -3} f(x)$     e)  $\lim_{x \rightarrow -1} f(x)$

2. Consider the following piece-wise defined function:  $f(x) = \begin{cases} 3 + 2x + x^2 & \text{if } x < 2 \\ 6 + 2x + x^2 & \text{if } x \geq 2 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow 2^-} f(x)$     b)  $\lim_{x \rightarrow 2^+} f(x)$     c)  $\lim_{x \rightarrow 2} f(x)$     d)  $\lim_{x \rightarrow 1} f(x)$     e)  $\lim_{x \rightarrow 3} f(x)$

3. Consider the following piece-wise defined function:  $f(x) = \begin{cases} -1 + 2x + 2x^2 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 6 & \text{if } x > 1 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow 1^-} f(x)$     b)  $\lim_{x \rightarrow 1^+} f(x)$     c)  $\lim_{x \rightarrow 1} f(x)$     d)  $\lim_{x \rightarrow 0} f(x)$     e)  $\lim_{x \rightarrow 2} f(x)$

4. Consider the following piece-wise defined function:  $f(x) = \begin{cases} -3 + 2x - x^2 & \text{if } x < -1 \\ -3 + 2x - x^2 & \text{if } x \geq -1 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow -1^-} f(x)$     b)  $\lim_{x \rightarrow -1^+} f(x)$     c)  $\lim_{x \rightarrow -1} f(x)$     d)  $\lim_{x \rightarrow -2} f(x)$     e)  $\lim_{x \rightarrow 0} f(x)$

5. Consider the following piece-wise defined function:  $f(x) = \begin{cases} 3 & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 8 - x - x^2 & \text{if } x > 1 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow 1^-} f(x)$     b)  $\lim_{x \rightarrow 1^+} f(x)$     c)  $\lim_{x \rightarrow 1} f(x)$     d)  $\lim_{x \rightarrow 0} f(x)$     e)  $\lim_{x \rightarrow 2} f(x)$

6. Consider the following piece-wise defined function:  $f(x) = \begin{cases} -4 + 2x + x^2 & \text{if } x < -2 \\ -15 - 3x + 2x^2 & \text{if } x > -2 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow -2^-} f(x)$     b)  $\lim_{x \rightarrow -2^+} f(x)$     c)  $\lim_{x \rightarrow -2} f(x)$     d)  $\lim_{x \rightarrow -3} f(x)$     e)  $\lim_{x \rightarrow -1} f(x)$

7. Consider the following piece-wise defined function:  $f(x) = \begin{cases} 2 - x - 2x^2 & \text{if } x < 3 \\ -10 & \text{if } x = 3 \\ -37 + x + 2x^2 & \text{if } x > 3 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow 3^-} f(x)$     b)  $\lim_{x \rightarrow 3^+} f(x)$     c)  $\lim_{x \rightarrow 3} f(x)$     d)  $\lim_{x \rightarrow 2} f(x)$     e)  $\lim_{x \rightarrow 4} f(x)$

8. Consider the following piece-wise defined function:  $f(x) = \begin{cases} 1 + 2x + 2x^2 & \text{if } x \leq 1 \\ 9 + x - 2x^2 & \text{if } x > 1 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow 1^-} f(x)$     b)  $\lim_{x \rightarrow 1^+} f(x)$     c)  $\lim_{x \rightarrow 1} f(x)$     d)  $\lim_{x \rightarrow 0} f(x)$     e)  $\lim_{x \rightarrow 2} f(x)$

9. Consider the following piece-wise defined function:  $f(x) = \begin{cases} -2 + 3x & \text{if } x < 2 \\ 19 - 2x - 2x^2 & \text{if } x > 2 \end{cases}$

Find the following limits:

a)  $\lim_{x \rightarrow 2^-} f(x)$     b)  $\lim_{x \rightarrow 2^+} f(x)$     c)  $\lim_{x \rightarrow 2} f(x)$     d)  $\lim_{x \rightarrow 1} f(x)$     e)  $\lim_{x \rightarrow 3} f(x)$

10. Consider the following piece-wise defined function:  $f(x) = \begin{cases} 2 + x + 2x^2 & \text{if } x < 0 \\ 1 & \text{if } x = 0 \\ 2 + x + 2x^2 & \text{if } x > 0 \end{cases}$

Find the following limits:

- a)  $\lim_{x \rightarrow 0^-} f(x)$     b)  $\lim_{x \rightarrow 0^+} f(x)$     c)  $\lim_{x \rightarrow 0} f(x)$     d)  $\lim_{x \rightarrow -1} f(x)$     e)  $\lim_{x \rightarrow 1} f(x)$

ANSWERS:

1. (a) 15 (b) 15 (c) 15 (d) 28 (e) 21  
 2. (a) 11 (b) 14 (c) DNE (d) 6 (e) 21  
 3. (a) 3 (b) 6 (c) DNE (d) 1 (e) 6  
 4. (a) 6 (b) 9 (c) 9 (d) 6 (e) 11 - 3  
 5. (a) 3 (b) 6 (c) DNE (d) 3 (e) 2  
 6. (a) 4 (b) 1 (c) DNE (d) 1 (e) 01 -  
 7. (a) 19 (b) 91 - (c) DNE (d) 8 - (e) 1 -  
 8. (a) 5 (b) 8 (c) DNE (d) 1 (e) 3  
 9. (a) 4 (b) 7 (c) DNE (d) 1 (e) 5 -  
 10. (a) 2 (b) 2 (c) 2 (d) 3 (e) 5

Solutions:

1.

a)  $\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} (1 - 3x + 2x^2) = 1 - 3(-2) + 2(-2)^2 = 15$

b)  $\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^+} (25 + 3x - x^2) = 25 + 3(-2) - (-2)^2 = 15$

c)  $\lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$ . Therefore  $\lim_{x \rightarrow -2} f(x) = 15$

d)  $\lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} (1 - 3x + 2x^2) = 1 - 3(-3) + 2(-3)^2 = 28$

e)  $\lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} (25 + 3x - x^2) = 25 + 3(-1) - (-1)^2 = 21$

2.

a)  $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (3 + 2x + x^2) = 3 + 2(2) + (2)^2 = 11$

b)  $\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (6 + 2x + x^2) = 6 + 2(2) + (2)^2 = 14$

c)  $\lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x)$ . Therefore  $\lim_{x \rightarrow 2} f(x) = \text{DNE}$

d)  $\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (3 + 2x + x^2) = 3 + 2(1) + (1)^2 = 6$

e)  $\lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} (6 + 2x + x^2) = 6 + 2(3) + (3)^2 = 21$

3.

a)  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (-1 + 2x + 2x^2) = -1 + 2(1) + 2(1)^2 = 3$

b)  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (6) = 6 = 6$

c)  $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$ . Therefore  $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

d)  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (-1 + 2x + 2x^2) = -1 + 2(0) + 2(0)^2 = -1$

e)  $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (6) = 6 = 6$

4.

a)  $\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^-} (-3 + 2x - x^2) = -3 + 2(-1) - (-1)^2 = -6$

b)  $\lim_{x \rightarrow -1^+} f(x) = \lim_{x \rightarrow -1^+} (-3 + 2x - x^2) = -3 + 2(-1) - (-1)^2 = -6$

c)  $\lim_{x \rightarrow -1^-} f(x) \neq \lim_{x \rightarrow -1^+} f(x)$ . Therefore  $\lim_{x \rightarrow -1} f(x) = -6$

d)  $\lim_{x \rightarrow -2} f(x) = \lim_{x \rightarrow -2} (-3 + 2x - x^2) = -3 + 2(-2) - (-2)^2 = -11$

e)  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (-3 + 2x - x^2) = -3 + 2(0) - (0)^2 = -3$

5.

a)  $\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (3) = 3 = 3$

b)  $\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (8 - x - x^2) = 8 - (1) - (1)^2 = 6$

c)  $\lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x)$ . Therefore  $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

d)  $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (3) = 3 = 3$

e)  $\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (8 - x - x^2) = 8 - (2) - (2)^2 = 2$

6.

a)  $\lim_{x \rightarrow -2^-} f(x) = \lim_{x \rightarrow -2^-} (-4 + 2x + x^2) = -4 + 2(-2) + (-2)^2 = -4$

b)  $\lim_{x \rightarrow -2^+} f(x) = \lim_{x \rightarrow -2^+} (-15 - 3x + 2x^2) = -15 - 3(-2) + 2(-2)^2 = -1$

c)  $\lim_{x \rightarrow -2^-} f(x) \neq \lim_{x \rightarrow -2^+} f(x)$ . Therefore  $\lim_{x \rightarrow -2} f(x) = \text{DNE}$

d)  $\lim_{x \rightarrow -3} f(x) = \lim_{x \rightarrow -3} (-4 + 2x + x^2) = -4 + 2(-3) + (-3)^2 = -1$

$$e) \lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} (-15 - 3x + 2x^2) = -15 - 3(-1) + 2(-1)^2 = -10$$

7.

$$a) \lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^-} (2 - x - 2x^2) = 2 - (3) - 2(3)^2 = -19$$

$$b) \lim_{x \rightarrow 3^+} f(x) = \lim_{x \rightarrow 3^+} (-37 + x + 2x^2) = -37 + (3) + 2(3)^2 = -16$$

$$c) \lim_{x \rightarrow 3^-} f(x) \neq \lim_{x \rightarrow 3^+} f(x) . \text{ Therefore } \lim_{x \rightarrow 3} f(x) = \text{DNE}$$

$$d) \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (2 - x - 2x^2) = 2 - (2) - 2(2)^2 = -8$$

$$e) \lim_{x \rightarrow 4} f(x) = \lim_{x \rightarrow 4} (-37 + x + 2x^2) = -37 + (4) + 2(4)^2 = -1$$

8.

$$a) \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} (1 + 2x + 2x^2) = 1 + 2(1) + 2(1)^2 = 5$$

$$b) \lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1^+} (9 + x - 2x^2) = 9 + (1) - 2(1)^2 = 8$$

$$c) \lim_{x \rightarrow 1^-} f(x) \neq \lim_{x \rightarrow 1^+} f(x) . \text{ Therefore } \lim_{x \rightarrow 1} f(x) = \text{DNE}$$

$$d) \lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} (1 + 2x + 2x^2) = 1 + 2(0) + 2(0)^2 = 1$$

$$e) \lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} (9 + x - 2x^2) = 9 + (2) - 2(2)^2 = 3$$

9.

$$a) \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (-2 + 3x) = -2 + 3(2) = 4$$

$$b) \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (19 - 2x - 2x^2) = 19 - 2(2) - 2(2)^2 = 7$$

$$c) \lim_{x \rightarrow 2^-} f(x) \neq \lim_{x \rightarrow 2^+} f(x) . \text{ Therefore } \lim_{x \rightarrow 2} f(x) = \text{DNE}$$

$$d) \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (-2 + 3x) = -2 + 3(1) = 1$$

$$e) \lim_{x \rightarrow 3} f(x) = \lim_{x \rightarrow 3} (19 - 2x - 2x^2) = 19 - 2(3) - 2(3)^2 = -5$$

10.

$$a) \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} (2 + x + 2x^2) = 2 + (0) + 2(0)^2 = 2$$

$$b) \lim_{x \rightarrow 0^+} f(x) = \lim_{x \rightarrow 0^+} (2 + x + 2x^2) = 2 + (0) + 2(0)^2 = 2$$

$$c) \lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^+} f(x) . \text{ Therefore } \lim_{x \rightarrow 0} f(x) = 2$$

$$d) \lim_{x \rightarrow -1} f(x) = \lim_{x \rightarrow -1} (2 + x + 2x^2) = 2 + (-1) + 2(-1)^2 = 3$$

$$e) \lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} (2 + x + 2x^2) = 2 + (1) + 2(1)^2 = 5$$